Institute of Physics
Centre for Science and Education
Silesian University of Technology

Physics laboratory report

Performed on: 04.11.2024

Exercise 1: Determination of the acceleration due to gravity using a simple pendulum

Performed by:

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Introduction

The experiment investigates the relationship between the period of a simple pendulum and its length to determine Earth's gravitational acceleration g. By analyzing the pendulum's motion, we aim to derive a value for gg and compare it with theoretical predictions based on our location. This study demonstrates how basic principles of physics can be applied to measure fundamental constants of nature.

Experimental set-up

The experimental set-up is shown in Figure 1. The pendulum is fixed on a column mounted on the base of the device. The length of the pendulum can be changed using a dial. The length can be taken from the column using the white strip on the bob. The timer uses a light cell mounted on a holder whose position can be changed. The measurement consists of reading the time of N beats as a function of the length of the pendulum. In our case the amount of beats given by instructor was $N = 10$.

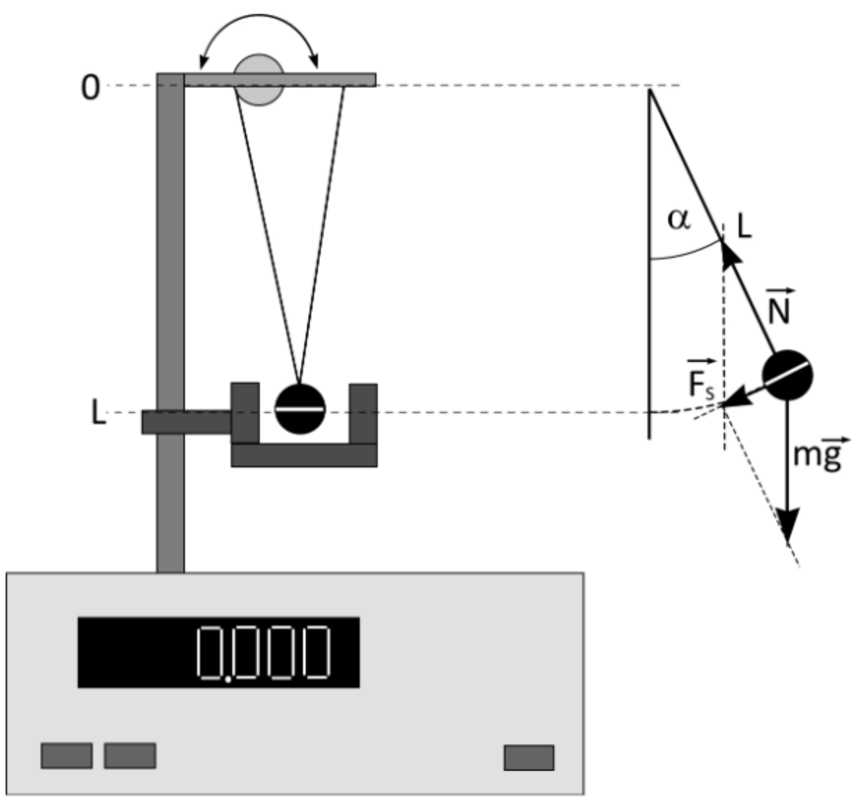

Figure 1 - The experimental set-up

Measurements

Lp.	L (cm)	t_1 (s)	t_2 (s)	t_3 (s)	t_4 (s)	t_5 (s)
1	10	6.22	6.21	6.22	6.21	6.20
2	20	8.89	8.90	8.90	8.89	8.90
3	30	10.88	10.87	10.85	10.86	10.88
4	40	12.64	12.64	12.65	12.65	12.65
5	50	14.12	14.10	14.10	14.10	14.09
6	60	15.45	15.42	15.45	15.44	15.45
7	70	16.70	16.69	16.70	16.70	16.75
8	80	17.85	17.85	17.86	17.87	17.87
9	90	18.93	18.93	18.93	18.93	18.93

Table 1 - L is the pendulum length in cm, and t_1 to t_5 are the times recorded for $N = 10$ oscillations.

Data analysis

Task 1. or each length of the pendulum calculate the values of $\sqrt{L}$ and average values of the measured time of N beats.

To obtain the average value of the measured times we used formula:

$$t_{sr} = \frac{1}{n} \sum_{i=1}^{n} t_i \ [s]$$

As we repeated measurements 5 times for each L, $n = 5$.

Task 2. Calculate the statistical uncertainty $u_a(t_{sr})$, as the standard deviation of the mean multiplied by the appropriate Student-Fischer coefficient.

To calculate the statistical uncertainty we used formula:

$$u_a(t_{sr}) = sk \; [s]$$

where:

- s is standard deviation mean calculated from equation:

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(t_i - t_{sr})^2}$$

- and k is Student-Fischer coefficient. From table we read that for confidence level 95% and 5 measurements we read that the coefficient $k \approx 2.776$.

Task 3. For each length of the pendulum calculate its $T = \frac{t_{sr}}{N}$

The unit for the result of the equation is in s and $N = 10$.

Task 4. Using the uncertainty propagation law calculate the uncertainties of the determined periods.

To calculate the uncertainty of period using uncertainty propagation law we used the formula:

$$u(T) = \frac{u_a(t_{sr})}{N} \; [s]$$

where:

- $u_a(t_{sr})$ is previously calculated statistical uncertainty in average time
- N is number ob beats

Task 5. Note the results in the table.

Lp.	L (m)	$\sqrt{L}$ ($\sqrt{\mathbf{m}}$)	t_{sr} (s)	$u_a(t_{sr})$ (s)	T (s)	$u(T)$ (s)
1	0.1	0.3162	6.2120	0.0104	0.6212	0.0010
2	0.2	0.4472	8.8960	0.0068	0.8896	0.0007
3	0.3	0.5477	10.8680	0.0162	1.0868	0.0016
4	0.4	0.6325	12.6460	0.0068	1.2646	0.0007
5	0.5	0.7071	14.1020	0.0136	1.4102	0.0014
6	0.6	0.7746	15.4420	0.0162	1.5442	0.0016
7	0.7	0.8367	16.7080	0.0296	1.6708	0.0030
8	0.8	0.8944	17.8600	0.0124	1.7860	0.0012
9	0.9	0.9487	18.9300	0.0000	1.8930	0.0000

Table 2 - L is the pendulum length in m, $\sqrt{L}$ is the square root of the length in $\sqrt{m}$, t_{sr} is the average time in s, $u_a(t_{sr})$ is statistical uncertainty of the average time in s, T is period in s and $u(T)$ is uncertainty of the period in s.

Task 6. Plot the dependence of $T(L)$ including the error bars.

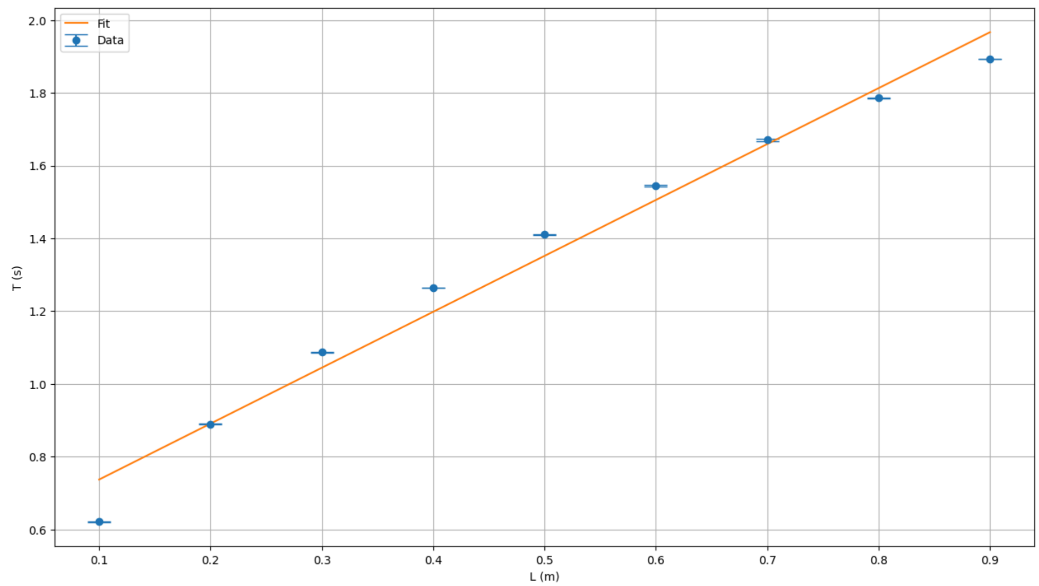

Figure 2 - plot of the dependence of $T(L)$ with error bars and fitted function.

Task 7. Plot the dependence $T(\sqrt{L})$ including the error bars.

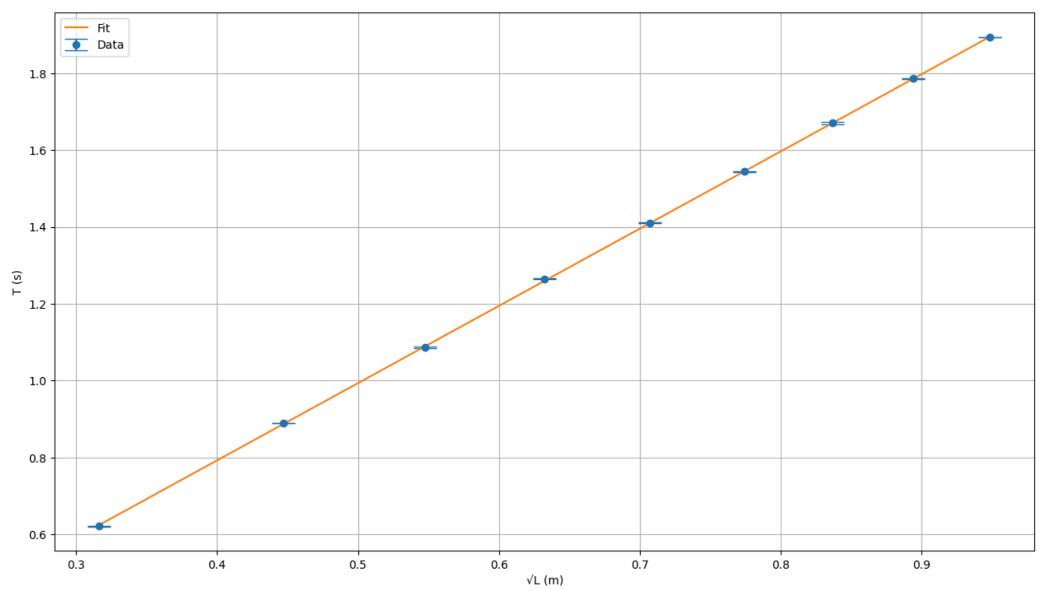

Figure 3 - plot of the dependence of $T(\sqrt{L})$ with error bars and fitted function.

Task .8 Using linear regression determine the coefficients of the straight line $T(\sqrt{L})$ and their standard uncertainties. Plot the line on the graph. Does the line pass outside the error bars?

To calculate the linear regression we used python library scipy and its linregress function to obtain the results and omit tons of calculations. The code used to obtain solutions is placed bellow:

```
from scipy.stats import linregress
import numpy as np

def task_8(sqrt_l, periods, uncertainties_of_the_period):
    sqrt_l = np.array(sqrt_l)
    periods = np.array(periods)
    uncertainties_of_the_period = np.array (uncertainties_of_the_period)
    slope, x, y, z, std_err = linregress(sqrt_l, periods)
    return slope, std_err
```

To the function we passing the list of square roots of L's, list of T's and list of $u(T)$'s. The function returns the parameter m (`slope`) and standard uncertainty (`std_err`).

The function from library finds the best fitting parameters m and c for linear function:

$$y = mx + c$$

The plotted fit is visible on Figure 3. The slope we obtained have parameter $m = 2.0099$ and standard uncertainty equal to 0.0045. We notice the line stays within the error bars, which means regression model accurately represents the relationship within the uncertainty range, indicating a good fit.

Task 9. Using the slope of the line and using the equation for the period of the pendulum calculate the gravitational acceleration g.

To calculate the gravitational acceleration using the equation for the period we used formula:

$$T = 2\pi\sqrt{\frac{L}{g}}\,[s] \Rightarrow g = \frac{4\pi^2 L}{T^2}\,\left[\frac{m}{s^2}\right]$$

From this equation we can go further and apply the slope of the line $T(\sqrt{L})$ and calculate g via:

$$g = \frac{4\pi^2}{m^2}\,\left[\frac{m}{s^2}\right]$$

After putting numbers into equation we obtain $g = 9.7728\,[\frac{m}{s^2}]$.

Task 10. Using the uncertainty propagation law calculate the uncertainties of the determined g .

To calculate the uncertainty we used formula:

$$u(g) = g\frac{2u_{st}}{m} \quad (1) \quad \left[\frac{m}{s^2}\right]$$

where:

- u_{st} is standard uncertainty
- m is coefficients of the straight line $T(\sqrt{L})$

which was obtained using propagation law:

$$u(g) = g\left|\frac{\partial}{\partial m}\left(\frac{4\pi^2}{m^2}\right)\right|u_{st}$$

From partial derivative we obtain:

$$\frac{\partial g}{\partial m} = -\frac{8\pi^2}{m^3}$$

The sign does not matter since we only need absolute value of the derivative. Next from the equation we can see that g depends on $\frac{1}{m^2}$ therefore the relative uncertainty in g is proportional to $\frac{2u_{st}}{m}$.

from which we obtain the final equation$_{(1)}$.

From calculations we obtained $u(g) = 0.044 \left[\frac{m}{s^2}\right]$.

Task 11. Calculate the expanded uncertainty.

To calculate the expended uncertainty we used formula:

$$U = k \cdot u(g) \left[\frac{m}{s^2}\right]$$

where k is coverage factor which we read from tables and is equal 2 and $u(g)$ is uncertainty in g calculated previously.

From calculations we obtained $U = 0.088 \left[\frac{m}{s^2}\right]$.

Task 12. Perform a statistical test of agreement of the obtained value with the value calculated for your latitude and height above sea level (Gliwice).

We used formula:

$$Z = \frac{|Z_{measured} - Z_{theoretical}|}{u(g)}$$

For 95% confidence level the $Z_{critical} \approx 1.96$.

To check the theoretical value with respect to latitude we used formula:

$$g(\lambda, h) = g_0(1 + f(\lambda) - \frac{2h}{R})$$

where:

- $g(\lambda, h)$ is the gravitational acceleration at latitude λ and altitude h
- $g_0 \approx 9.780318 \frac{m}{s^2}$ is the standard gravitational acceleration at sea level (at the equator)
- λ is the latitude in radians
- h is is the height above sea level equal to $200\ m$
- $R \approx 6378137m$ is the average radius of the Earth
- $f(\lambda)$ is the correction factor for latitude, calculated as:

$$f(\lambda) = (\frac{1}{focal\ distance} - \frac{1}{equatorialdistance}) \cos^2(\lambda)$$

where:

- the $focal\ distance$ is R - flattening factor
- the $equatorial\ distance$ is the radius of the Earth at the equator.

And for the practicality we used simplified version of the formula $g(\lambda, h)$:

$$g(\lambda, h) = g_0(1 + 0.0053024 \sin^2(\lambda) - 0.0000058 \sin^2(2\lambda) - \frac{2h}{R})$$

For the calculations we checked latitude in Gliwice which is approximately 50.3° and the elevation is approximately 200 meters.

Next we calculated the latitude in radians:

$$\lambda = 50.3^\circ \times \frac{\pi}{180^\circ} \approx 0.876\ radians$$

so we performed calculations:

$$g(50.3^{\circ}, 200\ m) \approx 9.8103 \frac{m}{s^2}$$

Putting theoretical value to the equation Z we obtained:

$$Z = 0.85$$

As the $0.85 < Z_{critical} = 1.96$, the measured value is consistent with the theoretical value, but might indicate the consistent error in time measurement.

Summary

In this experiment, we measured the period of a pendulum for various lengths and analyzed the data to calculate the gravitational acceleration at our location. Using linear regression, we determined the slope of the $T(L)$ graph, which allowed us to calculate the value of gravitational acceleration as $g = 9.7728\ \frac{m}{s^2}$. The uncertainty in g was found to be $u(g) = 0.044\ \frac{m}{s^2}$, with an expanded uncertainty of $U = 0.088\ \frac{m}{s^2}$. The statistical test of agreement showed that our measured value is consistent with the theoretical value for Gliwice, which means that measuring set-up was precise enough to obtain reliable result.

Measurement cards

Physics laboratories : Pendulum

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liczba okresów : 10

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	100					