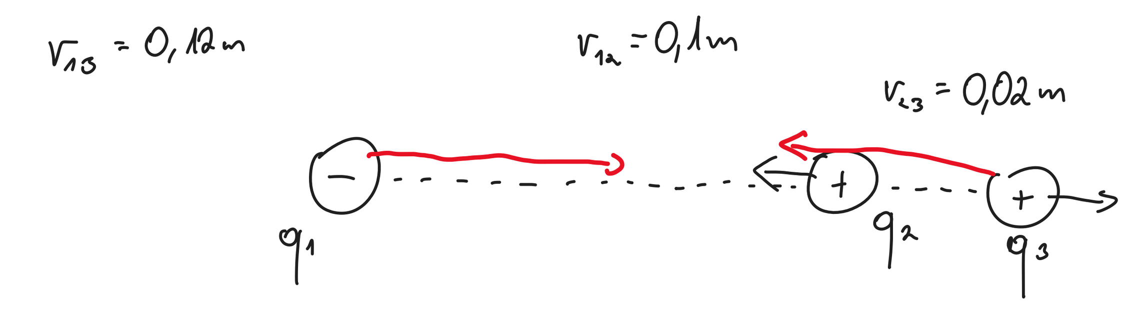
Problem 7.1. Two point charges $q_1 = -10^{-8}$ C and $q_2 = +1.5 \cdot 10^{-8}$ C are located at a distance $r_{12} = 10$ cm from each other. Find the force acting on a point charge $q_3 = 0.1 \cdot 10^{-8}$ C placed on the extension of the segment r_{12}, at a distance $r_{23} = 2$ cm from the charge q_2.
Ans. $3.31 \cdot 10^{-4}$ N.

$q_1 \; q_2 \; q_3$ $\qquad F = k\frac{qQ}{r^2}$



$F_{13} = k\frac{q_1 q_3}{r_{13}^2}$

$F_{23} = k\frac{q_2 q_3}{r_{23}^2}$

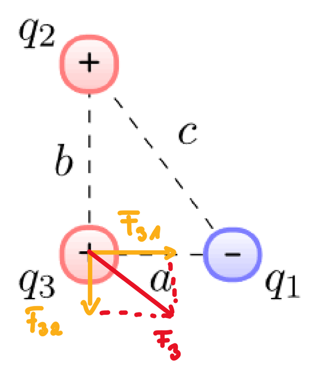
$F_{23} - F_{13}$

$$9 \cdot 10^9 \cdot \frac{-10^{-8} \cdot 0{,}1 \cdot 10^{-8}}{(0{,}12)^2} = -6{,}25 \cdot 10^{-6} = F_{13}$$

$$9 \cdot 10^9 \cdot \frac{1{,}5 \cdot 10^{-8} \cdot 0{,}1 \cdot 10^{-8}}{0{,}02^2} = 3{,}375 \cdot 10^{-4} = F_{23}$$

$$|F_{23} + F_{13}| = |3{,}375 \cdot 10^{-4} - 6{,}25 \cdot 10^{-6}| \approx 3{,}3125 \cdot 10^{-4}$$

Problem 7.2. At the vertices of a right-angled triangle with legs $a = 3$ m and $b = 4$ m there are charges $q_1 = -4 \cdot 10^{-4}$ C, $q_2 = 4 \cdot 10^{-4}$ C, and $q_3 = 0.8 \cdot 10^{-4}$ C. What is the force acting on the charge q_3?



$F_{32} = k\frac{q_3 q_2}{b^2} \qquad F_{31} = k\frac{q_3 q_1}{a^2} \qquad F_3 = \sqrt{F_{32}^2 + F_{31}^2}$

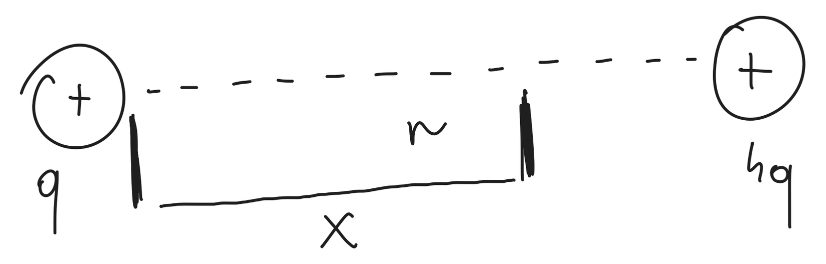
$$|F_{32}| = k\frac{0{,}8 \cdot 10^{-4} \cdot 4 \cdot 10^{-4}}{4^2} = \frac{3{,}2 \cdot 10^{-8}}{16} = \frac{32 \cdot 10^{-9}}{16} = 2 \cdot 10^{-9} k = 9 \cdot 2 = 18$$

$$|F_{31}| = k\frac{|0{,}8 \cdot 10^{-4} \cdot (-4 \cdot 10^{-4})|}{9} = \frac{3{,}2 \cdot 10^{-8}}{9} = 3{,}556 \cdot 10^{-9} k = 9 \cdot 3{,}556 \approx 32$$

$$F = \sqrt{18^2 + 32^2} = \sqrt{324 + 1024} \approx 36{,}7\,N$$

Problem 7.3. Two positive point charges q and $4q$ are located at a distance r from each other. What is the closest distance from the charge q at which the electric field intensity is zero? Calculate the potential at this point.

$E = k\frac{q}{d^2}$ electric field $\qquad V = k\frac{q}{d}$ electric field



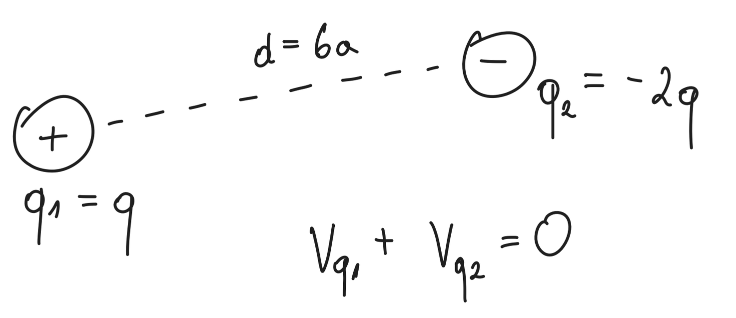
$E_q = E_{4q}$

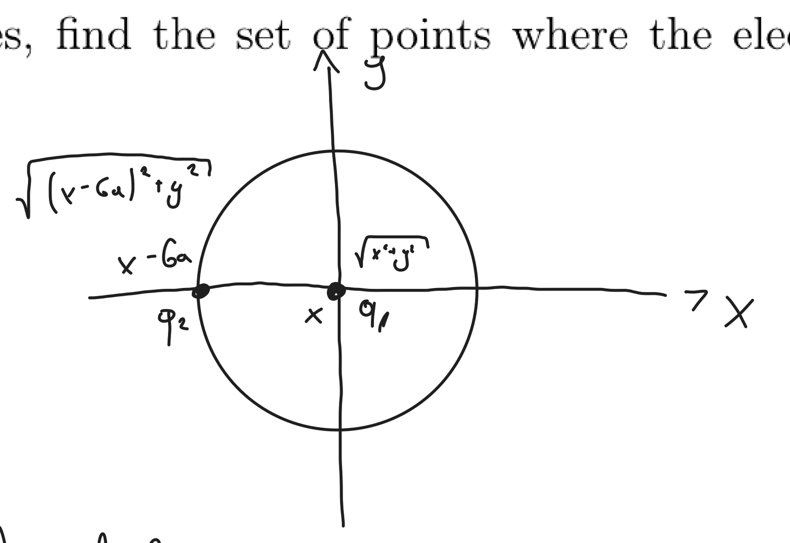
$$\frac{kq}{x^2} = \frac{k4q}{(r-x)^2} \Rightarrow x^2 = \frac{(r-x)^2}{4} \qquad 2x = r - x \qquad 3x = r \qquad x = \frac{r}{3}$$

$$V_q = k\frac{q}{\frac{r}{3}} = \frac{3kq}{r} \qquad V_{4q} = k\frac{4q}{\frac{2}{3}r} = \frac{6kq}{r}$$

$$V = V_q + V_{4q} = \frac{9kq}{r}$$

Problem 7.4. Two electric charges $q_1 = q$ and $q_2 = -2q$ are located at a distance $d = 6a$ from each other. On the plane containing these two charges, find the set of points where the electric potential is zero.



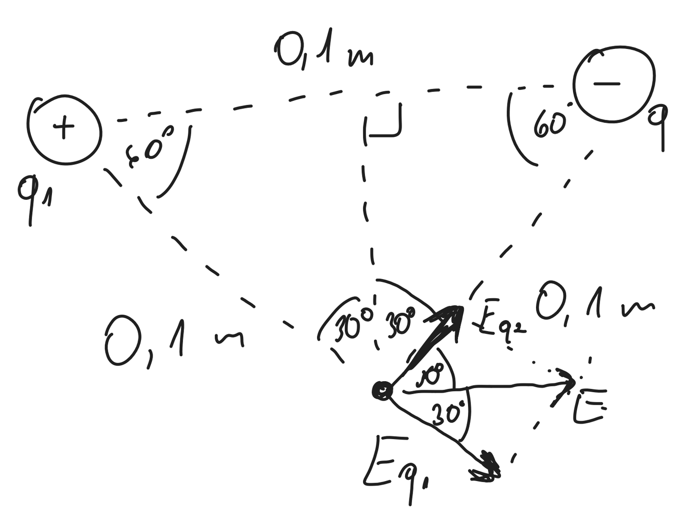


$V_{q_1} + V_{q_2} = 0$

$$\frac{q}{\sqrt{x^2 + y^2}} = \frac{2q}{\sqrt{(x-6a)^2 + y^2}} \quad \Big| \cdot \frac{1}{2} \Big|^{-2}$$

$$4(x^2 + y^2) = (x - 6a)^2 + y^2$$
$$4x^2 + 4y^2 = x^2 - 12ax + 36a^2 + y^2$$
$$3x^2 + 3y^2 + 12ax = 36a^2 \quad | : 3$$
$$x^2 + y^2 + 4ax - 12a^2 = 0$$
$$x^2 + 4ax + 4a^2 + y^2 - 16a^2 = 0$$
$$(x + 2a)^2 + y^2 = (4a)^2$$

Problem 7.5. The electrostatic field is generated by two point charges $q_1 = +5 \cdot 10^{-9}$ C and $q_2 = -5 \cdot 10^{-9}$ C. The distance between the charges is $r = 10$ cm. Find the potential and the field intensity at a point 10 cm away from each charge. The permittivity of vacuum is $\varepsilon_0 = 8.8542 \cdot 10^{-12}$ F/m.

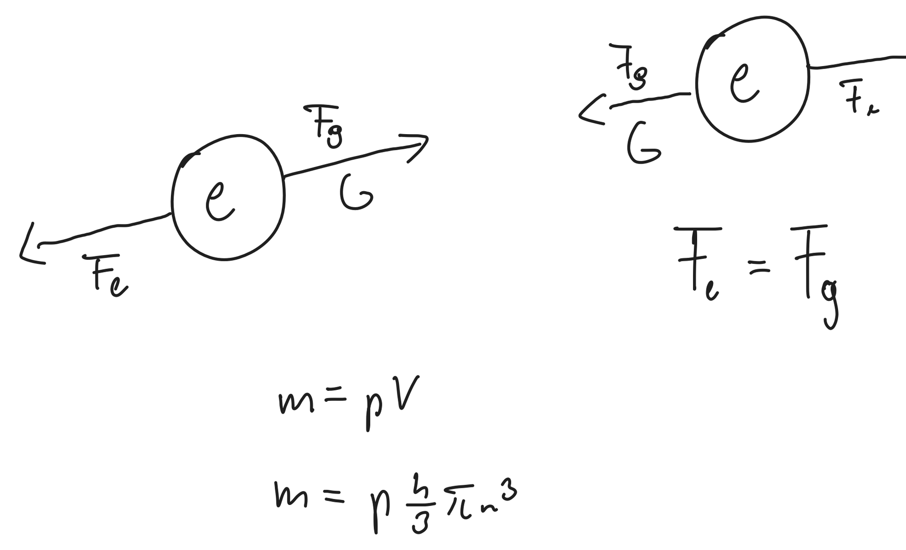


$V = k\frac{q}{d} \qquad E = k\frac{|q|}{d^2}$

$$V_{q_1} = k\frac{5 \cdot 10^{-9}}{0{,}1} \qquad V_{q_2} = k\frac{-5 \cdot 10^{-9}}{0{,}1} \qquad V = V_{q_1} + V_{q_2} = 0\,V$$

$$E = E_{q_1} \sin 30° + E_{q_2} \sin 30° = \frac{1}{2}(E_{q_1} + E_{q_2}) = 4{,}5 \cdot 10^3$$

Problem 7.6. Each of the two identical spherical water droplets carries a negative electric charge equal to the elementary charge $e = 1.602 \cdot 10^{-19}$ C. The electrostatic repulsion force is balanced by the gravitational attraction force. Calculate the radius of the droplet r. Gravitational constant $G = 6.674 \cdot 10^{-11}$ m^3/(kg·s^2), density of water $\rho = 997$ kg/m^3.



$F_e = F_g$

$F_e = k\frac{Qq}{d^2} \qquad F_g = G\frac{Mm}{d^2}$

$$k\frac{e^2}{r^2} = G\frac{m^2}{r^2}$$

$$k\frac{e^2}{r^2} = G\frac{\rho^2 \frac{16}{9}\pi^2 r^6}{r^2}$$

$m = \rho V$

$m = \rho \frac{4}{3}\pi r^3$

$$\sqrt[3]{\frac{3e\sqrt{\frac{k}{G}}}{4\rho\pi}} = r$$