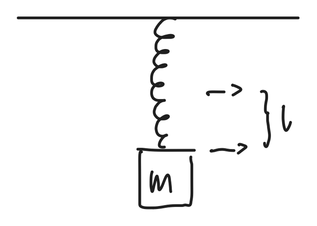
Problem 6.1. A weight was suspended on a vertically hanging spring, causing the spring to extend by length l. This weight was set into vibration by pulling it down and releasing it. What should be the value of the damping coefficient β so that (1) the vibrations cease after time t (assume that the vibrations cease when their amplitude decreases to 1% of the initial value); (2) the logarithmic decrement of damping is equal to $\Lambda = 6$.



$\beta = ?$

1) $0{,}01 A_0 = A_0 e^{-\beta t}$

$0{,}01 = e^{-\beta t}$

$\frac{1}{100} = \frac{1}{e^{\beta t}}$

$100 = e^{\beta t}$

$\frac{\ln 100}{t} = \beta$

$\Lambda = 6$

$\Lambda = \beta T$

$T = \frac{2\pi}{\omega}$

$\omega = \sqrt{\omega_0^2 - \beta^2}$

$\omega_0 = \sqrt{\frac{g}{l}}$

$\Lambda = \beta \frac{2\pi}{\sqrt{\omega_0^2 - \beta^2}}$

$\Lambda^2 = \beta^2 \frac{4\pi^2}{\omega_0^2 - \beta^2}$

$\Lambda^2(\omega_0^2 - \beta^2) = \beta^2 4\pi^2$

$\Lambda^2\omega_0^2 - \Lambda^2\beta^2 = \beta^2 4\pi^2$

$\Lambda^2\omega_0^2 = \beta^2(4\pi^2 + \Lambda^2)$

$\frac{\Lambda^2\omega_0^2}{4\pi^2 + \Lambda^2} = \beta^2$

$\frac{\omega_0^2}{\left(\frac{4\pi^2}{\Lambda^2} + 1\right)} = \beta^2$

$\beta = \sqrt{\frac{g}{l\left(\frac{4\pi^2}{\Lambda^2} + 1\right)}}$

Problem 6.2. The damped vibrations of a certain body are described by the equation $x(t) = A_0 e^{-\beta t}\sin(\omega t + \frac{\pi}{4})$. Calculate the velocity of this body at $t = 0$ and the times at which the extreme position is reached.

$v(t) = -A_0\beta e^{-\beta t}\sin\left(\omega t + \frac{\pi}{4}\right) + A_0\omega e^{-\beta t}\cos\left(\omega t + \frac{\pi}{4}\right)$

$v(0) = -A_0\beta\sin\left(\frac{\pi}{4}\right) + A_0\omega\cos\left(\frac{\pi}{4}\right)$

$v(0) = \frac{\sqrt{2}}{2}A_0(\omega - \beta)$

$0 = -A_0\beta e^{-\beta t}\sin\left(\omega t + \frac{\pi}{4}\right) + A_0\omega e^{-\beta t}\cos\left(\omega t + \frac{\pi}{4}\right)$

$0 = -\beta\sin\left(\omega t + \frac{\pi}{4}\right) + \omega\cos\left(\omega t + \frac{\pi}{4}\right)$

$0 = -\beta\tan\left(\omega t + \frac{\pi}{4}\right) + \omega$

$\frac{\omega}{\beta} = \tan\left(\omega t + \frac{\pi}{4}\right)$

$\omega t + \frac{\pi}{4} = \arctan\left(\frac{\omega}{\beta}\right)$

$t = \frac{\arctan\left(\frac{\omega}{\beta}\right) - \frac{\pi}{4}}{\omega}$

Problem 6.3. A block with a mass of $m = 1$ kg is attached to a spring with a spring constant of $k = 100$ N/m. The entire system is placed on a smooth table, and the end of the spring is fixed. Knowing that the system is in a medium with a resistance coefficient of $r = 2$ N·s/m, write the equations of motion and calculate: (1) frequency, (2) period of vibrations, (3) relaxation time τ, (4) damping coefficient β, (5) logarithmic decrement of damping Λ, (6) amplitude and initial phase if at $t = 0$ s the position $x_0 = 20$ cm and the velocity $v_0 = 2$ m/s, (7) displacement and velocity of vibrations at time $t = 2$ s assuming zero initial phase.

$m = 1\,kg$
$k = 100\,\frac{N}{m}$
$r = 2\,\frac{Ns}{m}$

1) $\omega_d = \omega_0\sqrt{1 - \zeta^2}$ ← damping ratio

$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{1}} = 10\,\frac{rad}{s}$

$\zeta = \frac{r}{2\sqrt{km}} = \frac{2}{2\sqrt{100}} = \frac{1}{10}$

$\omega_d = 10\sqrt{\frac{99}{100}}\,\frac{rad}{s}$

2) $T = \frac{2\pi}{\omega_d} = \frac{2\pi}{10\sqrt{\frac{99}{100}}}\,s$

3) $\tau = \frac{1}{\beta} = \frac{2m}{r} = \frac{2}{2} = 1\,s$

4) $\beta = \frac{2}{2} = 1\,\frac{1}{s}$

5) $\Lambda = \beta T = \frac{2\pi}{10\sqrt{\frac{99}{100}}}$

$\beta = \frac{r}{2m}$

6) $t = 0$ $\quad x_0 = 0{,}2\,m$ $\quad v_0 = 2\,\frac{m}{s}$

$A_0 = 5\cos\left(\arctan\left(\frac{10+\beta}{\omega_d}\right)\right)$

$\begin{cases} 0{,}2 = A_0 e^{-\beta t}\cos(\varphi_0) \\ 2 = -A_0\beta e^{-\beta t}\cos(\varphi_0) - A_0\omega_d e^{-\beta t}\sin(\varphi_0) \end{cases}$

$10 A_0\cos\varphi_0 = -A_0\beta\cos\varphi_0 - A_0\omega_d\sin\varphi_0 \quad | : \cos\varphi$

$10 = -\beta - \omega_d\tan\varphi_0$

$\frac{10+\beta}{\omega_d} = \tan\varphi_0 \qquad \arctan\left(\frac{10+\beta}{\omega_d}\right) = \varphi$

7) $x(2) = A_0 e^{-2\beta}\cos(2\omega_d)$

$v(2) = -A_0\beta e^{-2\beta}\cos(2\omega_d) - A_0\omega_d e^{-2\beta}\sin(2\omega_d)$

Problem 6.4. A certain body can perform damped harmonic vibrations with an angular frequency ω and a damping coefficient β. At a certain moment, the body, located at the equilibrium position, is given an initial velocity v_0. Determine the values of successive maximum distances of the body from the equilibrium position. For calculations assume: $\beta = 0.1$ m^{-1}, $\omega = 1$ rad/s, $v_0 = 10$ cm/s.

ω
β

$x(0) = 0$
$v(0) = 0{,}1\,\frac{m}{s}$

$\omega_d = 1\,\frac{rad}{s}$

$A_n = A_0 e^{-\beta T n}$

$A_n = 0{,}1 e^{-0{,}1 \cdot 2\pi n}$

$A_1 = 0{,}1 e^{-0{,}1 \cdot 2\pi \cdot 1}$

$A_2 = 0{,}1 e^{-0{,}1 \cdot 4\pi}$

$10\,\frac{cm}{s} \cdot \frac{0{,}01}{1} = 0{,}1\,\frac{m}{s}$

$0{,}1\,\frac{1}{m} \cdot \frac{1}{60} = \frac{1}{10} \cdot \frac{1}{60}\,\frac{1}{s} = \frac{1}{600}\,s^{-1}$

idk

$A_0 = 0{,}1\,m$

Problem 6.5. The amplitude A of damped vibrations decreased by half in $t_1 = 5$ minutes. In what time, counting from t_1, will the amplitude decrease eightfold in relation to the amplitude A.

A_0
$t_1 = 300\,s$
$\frac{1}{2}A_0$
$t_2 = x$
$\frac{1}{8}A_0$

$A(t) = 2A(t_1)$

$A_0 e^{-\beta t} = 2A_0 e^{-\beta t_1}$

$\frac{1}{2} = e^{-\beta t_1}$

$2 = e^{\beta t_1}$

$\ln(2) = \beta t_1$

$\frac{\ln(2)}{300} = \beta$

$A_0 e^{-\beta t} = 8A_0 e^{-\beta t_2}$

$\frac{1}{8} = e^{-\beta t_2}$

$8 = e^{\beta t_2}$

$\ln 8 = \frac{\ln(2)}{300} t_2$

$\frac{3\ln(2)}{\ln(2)} \cdot 300 = t_2$

$t_2 = 900\,s$

Problem 6.6. In a certain medium, a simple pendulum with mass m and length l performs vibrations with a logarithmic decrement of damping Λ_0. What will Λ be when the resistance of the medium r increases n times? How much should the resistance be increased for the pendulum to stop vibrating?

idk

6.4 idk