Problem 5.3 A body of mass $m = 0.5$ kg performs harmonic oscillations with an amplitude $A = 0.05$ m and frequency $f = 50$ Hz. Determine the maximum values of the restoring force F_z, potential energy E_p, kinetic energy E_k, velocity v, and acceleration a at the moment when the displacement $x = A/3$.

$F_z = -kx \rightarrow k = m\omega^2 \rightarrow 0{,}5 \cdot (100\pi)^2 = 5000\pi^2 \frac{N}{m}$

$\omega = 2\pi f$

$\omega = 2\pi \cdot 50 \frac{1}{s} = 100\pi \frac{rad}{s}$

$F_z = -5000\pi^2 \cdot \frac{0{,}05}{3}$

$F_z = -\frac{250\pi^2}{3} \approx -822{,}467 N$

$E_p = \frac{1}{2}kx^2$

$E_p = \frac{1}{2} 5000\pi^2 \cdot \left(\frac{0{,}05}{3}\right)^2$

$E_p = 2500\pi^2 \cdot \frac{0{,}0025}{9}$

$E_p = \frac{6{,}25\pi^2}{9} \approx 6{,}854 J$

$E_c = \frac{kA^2}{2}$

$\frac{kA^2}{2} = E_p + E_k$

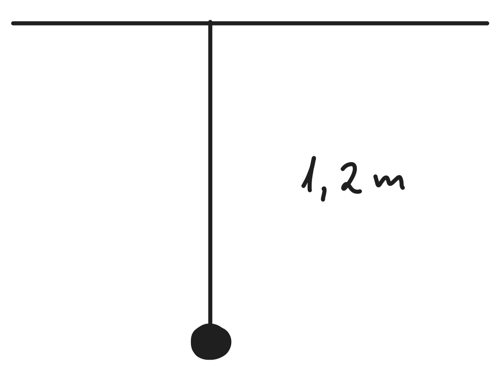
$\frac{5000\pi^2 \cdot 0{,}0025}{2} = 6{,}854 J + E_k$

$E_k = 54{,}83 J$

$v = \omega\sqrt{A^2 - x^2} = 100\pi\sqrt{0{,}05^2 - \left(\frac{0.05}{3}\right)^2} \approx 14{,}81 \frac{m}{s}$

$a = -\omega^2 x = -10000\pi^2 \cdot \frac{0{,}05}{3} \approx 1644{,}93 \frac{m}{s^2}$

Problem 5.4 A simple pendulum with a length of 120 cm hangs freely from the ceiling of a train car. What horizontal acceleration must the train have if the period of the pendulum's oscillation is 2.10 s?



$T = 2{,}1 s$

$T_0 = 2\pi\sqrt{\frac{L}{g_{eff}}}$

$g_{eff} = \sqrt{g^2 + a^2}$

$T_0 = 2\pi\sqrt{\frac{L}{\sqrt{g^2+a^2}}}$

$a = \pm 4{,}378$

Problem 5.5 A platform with a body lying on it can perform simple harmonic oscillations with an amplitude $A = 10$ cm in the horizontal direction. Determine the maximum frequency of oscillations at which the body will not slide if the coefficient of static friction between the platform and the body is $\mu_s = 0.45$.

$F_t = \mu_s mg$

$a_{max} = \omega^2 A$

$\omega = 2\pi f$

$f = \frac{\omega}{2\pi}$

$\cancel{m} a_{max} \leq \mu_s \cancel{m} g$

$\omega^2 A \leq \mu_s g$

$\omega \leq \sqrt{\frac{0{,}45 \cdot 10}{0{,}1}}$

$\omega \lessapprox 6{,}71$

$f = \frac{6{,}71}{2\pi} \approx 1{,}068$

Problem 5.6 A beaker is placed on a plate mounted on a spring such that the whole assembly can perform simple harmonic vertical motion with a period $T = 6$ s. When one drop of mercury falls into the beaker, the period of oscillation of the system becomes $T' = 7$ s. What would be the period of oscillation T'' if $n = 50$ identical drops of mercury fell into the beaker simultaneously?

$T = 2\pi\sqrt{\frac{m}{k}}$

$T = 6 s$

$T = 7 s$

$\begin{cases} T = 2\pi\sqrt{\frac{m}{k}} \\ T' = 2\pi\sqrt{\frac{m+\Delta m}{k}} \end{cases}$

$\frac{T'}{T} = \sqrt{\frac{m+\Delta m}{m}}$

$\left(\frac{7}{6}\right)^2 m - m = \Delta m$

$\frac{49-36}{36} m = \Delta m$

$\frac{13}{36} m = \Delta m$

$n = 50$

$n \cdot \Delta m = 50 \cdot \frac{13}{36} m = \frac{650}{36} m = \frac{325}{18} m$

$T'' = 2\pi\sqrt{\frac{\frac{343}{18}}{k}} = 2\pi\sqrt{\frac{343}{18}} \cdot \sqrt{\frac{m}{k}}$

$T'' = \sqrt{\frac{343}{18}}\, T$

$T'' = 6\sqrt{\frac{343}{18}}$

$T'' \approx 26{,}2$