Problem 5.1. A block of mass $m = 200$ g is attached to a spring with a spring constant $k = 5$ N/m. The system is placed on a smooth table, and the end of the spring is fixed. Knowing that at time $t = 2$ s the body was at position $x = 20$ cm and had a velocity $v = 2$ m/s, write the equations of motion and calculate: (1) amplitude, (2) initial phase, (3) frequency, (4) period of the oscillation, (5) maximum velocity of the body in this oscillatory motion. Ignore friction.

$x(t) = A\cos(\omega t + \alpha)$

$v(t) = -A\omega\sin(\omega t + \alpha)$

$$\begin{cases} 0{,}2 = A\cos(10 + \alpha) \quad | \cdot 10 \\ 2 = -5A\sin(10 + \alpha) \end{cases}$$

$\omega = \sqrt{\frac{k}{m}}$

$\omega = \sqrt{\frac{5}{0.2}} = 5$

1) $0{,}2 = A\cos(-1{,}1)$

$A = 0{,}441$ m

2) $10\cos(10 + \alpha) = -5A\sin(10 + \alpha)$

$-2 = \tan(10 + \alpha)$

$\alpha = -11{,}1$ rad

3) $f = \frac{\omega}{2\pi} = \frac{5}{2\pi} = 0{,}79$ Hz

4) $T = \frac{1}{f} = \frac{1}{0{,}79}$ s

5) $v_{max} = A\omega = 5 \cdot 0{,}441 = 2{,}205 \frac{m}{s}$

Problem 5.2 Calculate the frequency of undamped harmonic oscillations of a point mass of $m = 2$ g if the amplitude $A = 10$ cm, and the total energy of the oscillating point mass is 1 J.

$E_T = \frac{kA^2}{2}$

$1 = \frac{0{,}01 \cdot k}{2}$

$1 = 0{,}005\,k$

$k = 200$

$f = \frac{\omega}{2\pi}$ $\qquad f = 50{,}35$ Hz

$\omega = \sqrt{\frac{200}{0{,}002}}$ $\qquad T = 2\pi\sqrt{\frac{m}{k}}$

$\omega = 100\,000$