Problem 5.1. A block of mass $m = 200$ g is attached to a spring with a spring constant $k = 5$ N/m. The system is placed on a smooth table, and the end of the spring is fixed. Knowing that at time $t = 2$ s the body was at position $x = 20$ cm and had a velocity $v = 2$ m/s, write the equations of motion and calculate: (1) amplitude, (2) initial phase, (3) frequency, (4) period of the oscillation, (5) maximum velocity of the body in this oscillatory motion. Ignore friction.

$m = 0{,}2\ kg$

$k = 5 \frac{N}{m}$

$t = 2s$

$x = 0{,}2m$

$v = 2\frac{m}{s}$

1) ; 2)

$x(t) = A\cos(\omega t + \alpha)$

$v(t) = -\omega A\sin(\omega t + \alpha)$

$\omega = \sqrt{\frac{k}{m}}$

$\omega = \sqrt{\frac{5}{0{,}2}} = 5 \frac{rad}{s}$

$\rightarrow \begin{cases} 0{,}2 = A\cos(5\cdot 2 + \alpha) & | \cdot 10 \\ 2 = -5A\sin(10 + \alpha) \end{cases}$

$2 = 10A\cos(10+\alpha) = -5A\sin(10+\alpha)$

$-2 = \tan(10 + \alpha)$

$10 + \alpha = \arctan(-2)$

2) $\alpha \approx -11{,}107\ rad$

1) $A = \frac{0{,}2}{\cos(-1{,}107 rad)}$

$A = 0{,}447m$

3) $f = \frac{\omega}{2\pi} = \frac{5}{2\pi} = 0{,}796\ Hz$

4) $T = \frac{1}{f} = \frac{1}{0{,}796} = 1{,}256\ s$

5) $v_{max} = A\omega = 0{,}447 \cdot 5 = 2{,}235 \frac{m}{s}$

Problem 5.2 Calculate the frequency of undamped harmonic oscillations of a point mass of $m = 2$ g if the amplitude $A = 10$ cm, and the total energy of the oscillating point mass is 1 J.

$E_T = \frac{kA^2}{2}$

$f = \frac{\omega}{2\pi} \Rightarrow f = \frac{\sqrt{\frac{k}{m}}}{2\pi}$

$k = m\omega^2$

$\omega = \sqrt{\frac{k}{m}}$

$2E_T = kA^2$

$\frac{2E_T}{A^2} = k$

$\frac{2\cdot 1}{0{,}1^2} = k$

$k = 200$

$f = \frac{\sqrt{\frac{200}{0{,}002}}}{2\pi}$

$f = 50{,}329$

Problem 5.3 A body of mass $m = 0.5$ kg performs harmonic oscillations with an amplitude $A = 0.05$ m and frequency $f = 50$ Hz. Determine the maximum values of the restoring force F_z, potential energy E_p, kinetic energy E_k, velocity v, and acceleration a at the moment when the displacement $x = A/3$.

$m = 0{,}5kg$

$A = 0{,}05m$

$f = 50Hz$

$F_z = ?\quad E_p = ?\quad E_k = ?\quad$ IV $v = ?\quad$ V $a = ?$

$k = m\omega^2$

$x = \frac{A}{3} = \frac{0{,}05}{3}$

I $F_z = -kx$

II $E_p = \frac{kx^2}{2}$

III $E_k = E_T - E_p$, $E_T = \frac{kA^2}{2}$

$f = \frac{\omega}{2\pi}$

$f = \frac{\sqrt{\frac{k}{m}}}{2\pi}$

$2\pi f = \sqrt{\frac{k}{m}}$

$4\pi^2 f^2 m = k$

$k = 4\pi^2 50^2 \cdot 0{,}5$

$k = 49348{,}02 \frac{N}{m}$

I $F_z = -49348{,}02 \cdot \frac{0{,}05}{3} = 822{,}467$

II $E_p = \frac{49348{,}02 \cdot \left(\frac{0{,}05}{3}\right)^2}{2} = 6{,}853 J$

III $\frac{49348{,}02 \cdot 0{,}05^2}{2} - 6{,}853 = E_k$

$E_k = 54{,}832 J$

IV $v = \omega\sqrt{A^2 - x^2}$

$v = 2\pi f\sqrt{0{,}05^2 - \left(\frac{0{,}05}{3}\right)^2}$

$v = 100\pi\sqrt{0{,}05^2 - \left(\frac{0{,}05}{3}\right)^2}$

$v = 14{,}81 \frac{m}{s}$

V $a = -\omega^2 x$

$a = -(100\pi)^2 \cdot \frac{0{,}05}{3}$

$a = -1644{,}934 \frac{m}{s^2}$

Problem 5.4 A simple pendulum with a length of 120 cm hangs freely from the ceiling of a train car. What horizontal acceleration must the train have if the period of the pendulum's oscillation is 2.10 s?

$l = 1{,}2\ m$

$T_0 = 2{,}1s$

$T_0 = 2\pi\sqrt{\frac{L}{\sqrt{g^2 + a^2}}}$

$2{,}1 = 2\pi\sqrt{\frac{1{,}2}{\sqrt{100 + a^2}}}$

$\frac{4{,}41}{4\pi^2} = \frac{1{,}2}{\sqrt{100+a^2}}$

$4{,}41\sqrt{100+a^2} = 4\pi^2 \cdot 1{,}2$

$4{,}41\sqrt{100+a^2} = 47{,}374$

$19{,}448(100 + a^2) = 2244{,}305$

$1944{,}8 + 19{,}448a^2 = 2244{,}305$

$\pm 3{,}924 = a$

Problem 5.5 A platform with a body lying on it can perform simple harmonic oscillations with an amplitude $A = 10$ cm in the horizontal direction. Determine the maximum frequency of oscillations at which the body will not slide if the coefficient of static friction between the platform and the body is $\mu_s = 0.45$.

$F_f = \mu mg$

$a_{max} = A\omega^2$

$F_{net} = ma_{max}$

$f = \frac{\omega}{2\pi}$

$F_{net} \leq F_f$

$m a_{max} \leq \mu m g$

$A\omega^2 \leq \mu g$

$0{,}1\omega^2 = 0{,}45 \cdot 10$

$\omega^2 = \frac{4{,}5}{0{,}1}$

$\omega = \sqrt{\frac{4{,}5}{0{,}1}}$

$\omega = 6{,}708 \frac{rad}{s}$

$f = \frac{6{,}708}{2\pi}$

$f = 1{,}068\ Hz$

Problem 5.6 A beaker is placed on a plate mounted on a spring such that the whole assembly can perform simple harmonic vertical motion with a period $T = 6$ s. When one drop of mercury falls into the beaker, the period of oscillation of the system becomes $T' = 7$ s. What would be the period of oscillation T'' if $n = 50$ identical drops of mercury fell into the beaker simultaneously?

$T = 2\pi\sqrt{\frac{m}{k}}$

$\begin{cases} T = 6 \\ T' = 7 \\ T'' = ? \end{cases}$

$\begin{cases} 6 = 2\pi\sqrt{\frac{m}{k}} \\ 7 = 2\pi\sqrt{\frac{m+\Delta m}{k}} \end{cases}$

$\frac{6}{7} = \frac{2\pi\sqrt{\frac{m}{k}}}{2\pi\sqrt{\frac{m+\Delta m}{k}}}$

$\frac{6}{7} = \sqrt{\frac{m}{m+\Delta m}}$

$\frac{7}{6} = \sqrt{\frac{m+\Delta m}{m}}$

$\frac{49}{36} = \frac{m+\Delta m}{m}$

$\frac{49}{36}m = m + \Delta m$

$\frac{13}{36}m = \Delta m$

$n\cdot\Delta m = \frac{50\cdot 13}{36}m = \frac{650}{36}m = \frac{325}{18}m$

$T'' = 2\pi\sqrt{\frac{\frac{343}{18}m}{k}}$

$T'' = \sqrt{\frac{343}{18}}\, 2\pi\sqrt{\frac{m}{k}} \rightarrow T$

$T'' = \sqrt{\frac{343}{18}} \cdot 6$

$T'' \approx 26{,}192s$