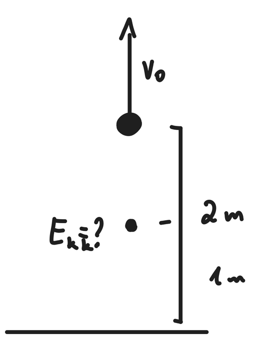
Problem 4.1. A body of mass $m = 100$ g was thrown vertically upwards from a height $h = 2$ m with an initial velocity $v_0 = 2$ m/s. Calculate the kinetic energy of this body at a height of 1 m.

$m = 100g = 0,1kg$

$h = 2m$

$v_0 = 2\frac{m}{s}$



$E_{k0} + E_{p0} = E_{kk} + E_{pk}$

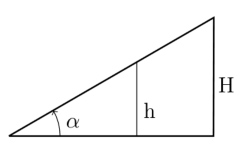
$\frac{mv_0^2}{2} + mgh = E_{kk} + mgh'$

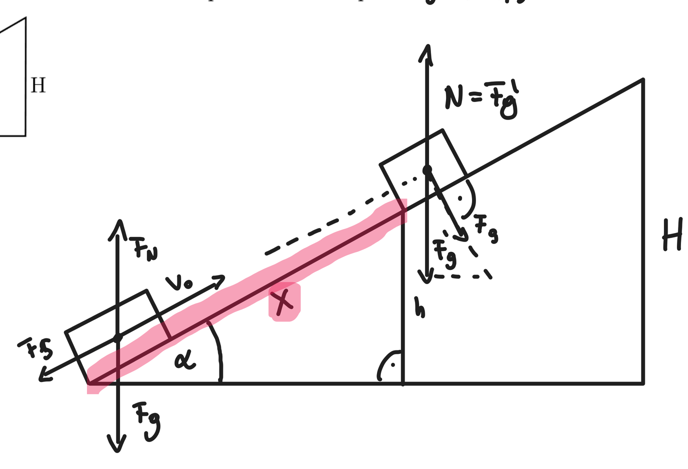
$0.1 \cdot 2 + 0.1 \cdot 10 \cdot 2 = E_{kk} + 0.1 \cdot 10 \cdot 1$

$0.2 + 2 = E_{kk} + 1$

$1.2 = E_{kk}$

Problem 4.2. Calculate at what height h will a block stop, pushed upwards on a slope with an angle of inclination $\alpha = 30°$ if the coefficient of friction of the block on the slope is $f = 0.24$. Assume that the block has small dimensions compared to the slope. $v_0 = 5m/s$





$F_s = \mu_s N$

$F_g' = F_g \cos\alpha$

$F_g' = mg\cos\alpha$

$\frac{mv^2}{2} + 0 = 0 + mgh + F_s x$

$\frac{mv^2}{2} = mgh + fmg\cos\alpha x$

$\frac{1}{2}v^2 = gh + gf\cos\alpha x$

$\frac{1}{2}v^2 = gh + ghf\cot\alpha$

$\frac{1}{2}v^2 = gh(1 + f\cot\alpha)$

$\frac{v^2}{2g(1+\cot(\alpha)f)} = h$

$E_k + E_p = E_k' + E_p' + F_f$

$\sin\alpha = \frac{h}{x}$

$x\sin\alpha = h$

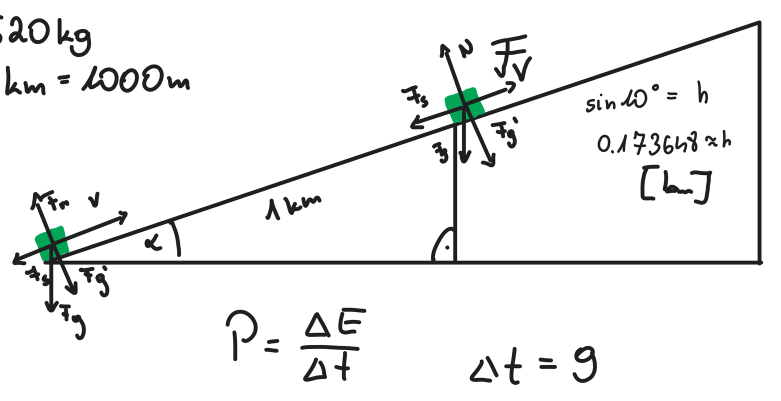
$x = \frac{h}{\sin\alpha}$

Problem 4.3. A horse pulls a sled up a hill which is inclined at an angle $\alpha = 10°$ to the horizontal. The sled, weighing $Q = 6200$ N, moves at a constant speed, covering 1 km in 9 minutes. With what power does the horse work pulling the sled? The coefficient of friction is $f = 0.05$. How will the required power from the horse change if the coefficient of friction doubles?

$t = 9min = 540s$

$m = 620kg$

$x = 1km = 1000m$



$Q = 6200N$

$\frac{kg\,m}{s^2}$

$F_{net} = ma$

$P = \frac{\Delta E}{\Delta t}$ $\Delta t = 9$

$N = mg$

$620\cancel{0} = m \cdot \cancel{10}$

$620kg = m$

Problem 4.4. A bullet of mass $m = 20$ g, moving at a speed $v = 600$ m/s, passes through a sandbag of mass $M = 150$ kg, causing the bag to move by $x = 20$ cm. The bullet's speed decreases by 5% after exiting the bag. Calculate the coefficient of friction of the bag on the ground, knowing that the friction of the bullet through sand is $T_{kp} = 300$ N (in this case, the force of resistance is proportional to speed), and the bag has a transverse size $s = 0.5$ m.

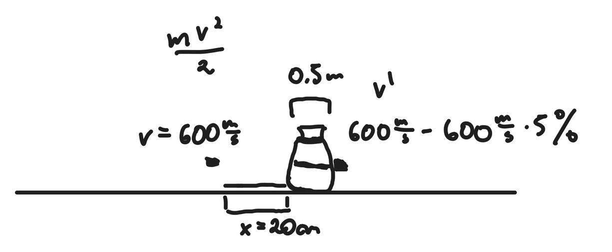
$(M+m)$

$f\,mg\,x$

$F_N = mg$

$F_f = f F_N$

$\frac{mv^2}{2}$



$E_{k0} = E_{kk}' + \underbrace{T_{kp} \cdot s}_{\text{Energy loss due to friction}} + \underbrace{f(m+M)gx}_{\text{Energy loss due to friction}}$

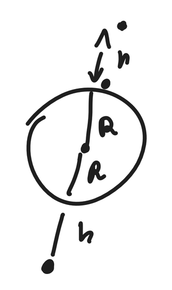
$E_k = E_k' + T_{kp} \cdot s + f(M+m)gx$

$\frac{mv^2}{2} = \frac{mv'^2}{2} + T_{kp}s + f(m+M)gx$

$mv^2 - mv'^2 - 2T_{kp}s + 2f(m+M)gx$

$\frac{mv^2 - mv'^2 - 2T_{kp}s}{2(m+M)gx} = f$

Problem 4.5. Calculate at what height above the Earth's surface the weight of a body is twice less than its weight on the surface of the Earth.



$F = G\frac{mM}{(R+h)^2}$

$F = G\frac{mM}{R^2}$

$F = G\frac{m_1 m_2}{r^2}$

$\frac{1}{2}\cancel{G}\frac{\cancel{mM}}{R^2} = \cancel{G}\frac{\cancel{mM}}{(R+h)^2}$

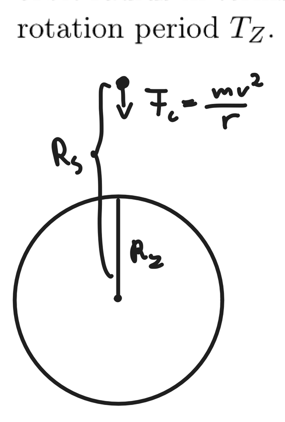
$\frac{1}{2R^2} = \frac{1}{(R+h)^2}$

$2R^2 = (R+h)^2$

$R\sqrt{2} = R+h$

$R\sqrt{2} - R = h$

Problem 4.6. At what distance R_S from the center of the Earth should a satellite orbit to remain constantly above the same point on the Earth's surface? Express the formula for the satellite's orbit radius in terms of the Earth's radius R_Z, the gravitational acceleration g, and the Earth's rotation period T_Z.



$R_s = (h + R_z)$

$F = G\frac{mM}{R_s^2}$

$\frac{mv^2}{r} = G\frac{mM}{r^2}$

$v^2 = \frac{GM}{r}$

$\left(\frac{2\pi r}{T_z}\right)^2 = \frac{GM}{r}$

$\frac{4\pi^2 r^2}{T_z^2} = \frac{GM}{r}$

$\frac{4\pi^2 r^3}{T_z^2} = GM$

$V_s = \frac{2\pi r}{T_z}$

$r^3 = \frac{GMT_z^2}{4\pi^2}$

$r = \sqrt[3]{\frac{GMT_z^2}{4\pi^2}}$

$r = \sqrt[3]{\frac{gR_z^2T_z^2}{4\pi^2}}$

$\frac{?}{t}$

$a = \frac{\Delta V}{\Delta t}$

$a = \frac{s}{t^2}$

$g = \frac{2\pi R_z}{T_z^2}$

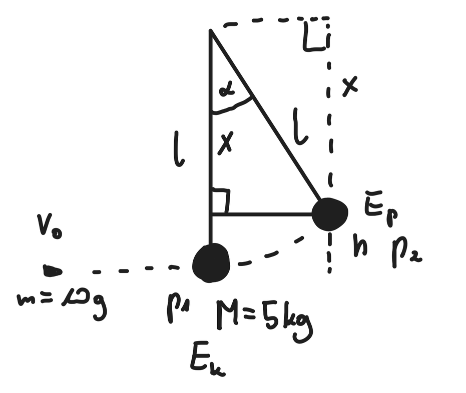
$mg = G\frac{mM}{R_z^2}$???

$\cancel{m}g = \frac{G\cancel{m}M}{R_z^2}$

$g = \frac{GM}{R_z^2}$

$\frac{gR_z^2}{M} = G$

Problem 4.7. To measure the speed of a bullet, we use what is known as a ballistic pendulum. It consists of a large mass body (a sandbag) suspended on a rigid rod. When the fired bullet is decelerated in the mass of the pendulum, its momentum is transferred to this mass. Calculate the bullet's speed v from the pendulum's deflection angle $\alpha = 31°$ and the length of the pendulum $l = 90$ cm, if the mass of the pendulum is $M = 5$ kg and the mass of the bullet is $m = 10$ g.



$\cos(31°) = \frac{x}{l}$

$x = 0.771451$

$0.9 - x = h$

$h = 0.128549$

$E_p = E_k$

$\cancel{(m+M)}gh = \frac{\cancel{(m+M)}v^2}{2}$

$v = \sqrt{2gh}$

$v = \sqrt{2 \cdot 10 \cdot h}$

$v = 1,603\frac{m}{s}$

$p_1 + p_2 = p_{system}$

$mv_0 + Mv_{null} = (m+M)v$

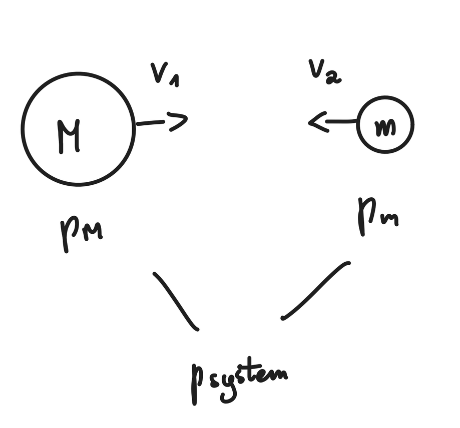
$v_0 = \frac{(m+M)v}{m}$

p_1 -> bullet momentum

p_2 -> pendulum momentum

$v_0 = \frac{(0.01+5) \cdot 1,603}{0.01} = 803,403\frac{m}{s}$

Problem 4.8. Two balls of masses $M = 5$ kg and $m = 3$ kg, moving at speeds $v_1 = 12$ cm/s and $v_2 = 4$ cm/s, collide head-on. Calculate the speeds of the balls after the collision: (a) in the case of inelastic balls, (b) in the case of perfectly elastic balls.



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