Problem 3.12. A flywheel initially making N revolutions per minute starts to slow down. Calculate the time after which the angular velocity of the wheel is halved, if the disc's motion is uniformly decelerated and the disc made x revolutions in this time.

Data:

N rpm $\quad \omega_0 \quad t_0 = 0 \quad R$

$t = ?$ $\quad x$ revolutions in time t $\quad [s(t)]$

$\omega_k = \frac{\omega_0}{2}$

$$rpm = \frac{2\pi}{60}\ \frac{rad}{s}$$

$$N\ rpm = \frac{2\pi N}{60}\ \frac{rad}{s}$$

$$v = \omega r$$

$$a = \frac{\Delta\omega}{\Delta t}$$

$$\Delta\omega = -\frac{1}{2}\omega_0 R$$

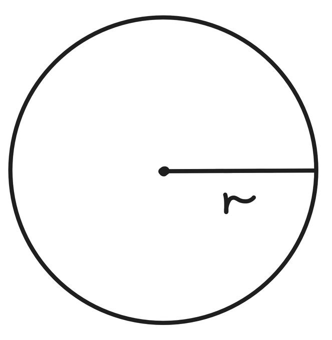
$$\Delta t = -t$$

$$s(t) = s_0 + \frac{at^2}{2}$$

$$x = \frac{\frac{\omega_0 R}{2\cancel{t}}\, t^{\cancel{2}}}{2} \Rightarrow x = \frac{\omega_0 R t}{4}$$

$$\frac{4x}{\omega_0 R} = t$$

Problem 3.13. Initially stationary, a disc of radius r is set into rotational motion. Find the angular velocity of the disc after time t_k, knowing that points on the rim of the disc covered a distance of $100\pi r$ during this time. Assume that the angular acceleration of the disc is constant.



Data: $t_0 = 0 \longrightarrow t_k$ $\quad \omega_k = ?$

$V_0 = 0 \quad v_k = ??$

$\omega_0 = 0 \quad d = 100\pi r$

$a = const$

$$a = \frac{\Delta\omega}{\Delta t} = \frac{\omega_k}{t_k}$$

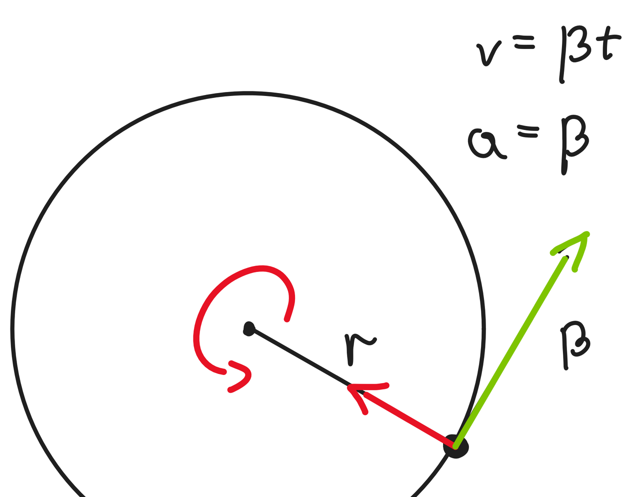
$$s(t) = s_0 + \frac{at^2}{2}$$

$$100\pi r = \frac{\frac{\omega_k}{\cancel{t_k}}\, t_k^{\cancel{2}}}{2}$$

$$200\pi r = \omega_k t_k$$

$$\frac{200\pi r}{t_k} = \omega_k$$

Problem 3.14. A point mass moves around a circle with a velocity $v = \beta t$, where $\beta = 0.50$ m/s^2. Find its acceleration and the angle it makes with the velocity vector after completing the first full rotation.



$v = \beta t$

$a = \beta$

$$2\pi r = \frac{\beta t^2}{2}$$

$$4\pi r = \beta t^2$$

$$t = \sqrt{\frac{4\pi r}{\beta}}$$

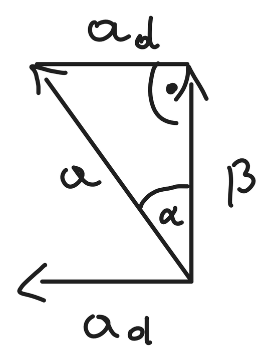
$$s(t) = \frac{at^2}{2}$$

$$a = \frac{\Delta\omega}{\Delta t} \Rightarrow \omega_k, \ \searrow t$$

$$a_c = \frac{v^2}{r} = \frac{\beta^{\cancel{2}}\,\frac{4\pi r}{\cancel{\beta}}}{r} = \frac{4\beta\pi \cancel{r}}{\cancel{r}} = 4\beta\pi$$

$$a = \sqrt{a_c^2 + \beta^2} = \sqrt{\beta^2 + (4\beta\pi)^2} =$$

$$= \beta\sqrt{1 + (4\pi)^2}$$



$$\tan\alpha = \frac{a_d}{\beta}$$

$$\tan\alpha = \frac{4\cancel{\beta}\pi}{\cancel{\beta}}$$

$$\alpha = \arctan(4\pi)$$

$$\alpha \approx 1{,}5\,rad$$

Problem 3.15. Calculate the linear velocity of a point in the Earth's rotational motion (in a system tied to the centre of the Earth): (A) at the equator, (B) at a latitude of $48°24'$. The radius of the Earth is 6378 km.

A) $v = \frac{s}{t}$ $\quad t = 24h$ $\quad s = 6378$

B) $v_A \cos(48°24') = v_B$

$$v_A = \frac{6378}{24}\ \frac{km}{h}$$

Problem 3.16. A particle covered half a circle of radius $R = 160$ m in a time $t = 10.0$ s with a constant value of tangential acceleration. Calculate over the time interval from 0 to t: (a) the magnitude of the average velocity vector; (b) the magnitude of the average acceleration vector; (c) the angle between these vectors.

Data:

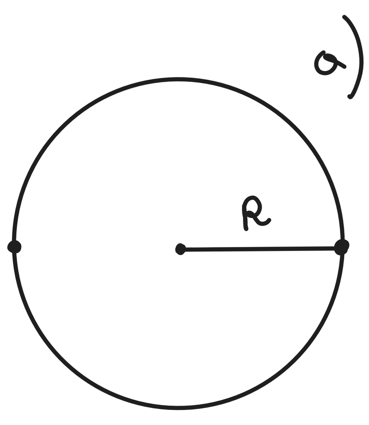
$R = 160m$

$d = 160\pi$

$t = 10s$

$a_t = const$

average velocity : $\frac{\text{displacement}}{\text{time}}$

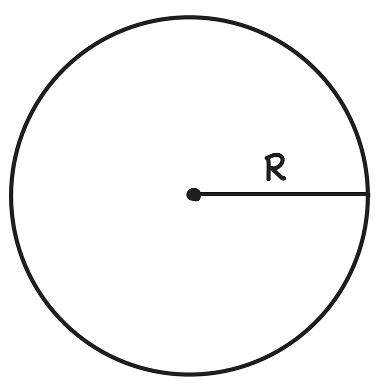


a) $V_{avg} = \frac{2R}{t} = \frac{320}{10} = 32\,\frac{m}{s}$

b) $a_{avg} = \frac{\Delta V}{\Delta t} = \frac{\frac{s}{t}}{t} = \frac{s}{t^2} = \frac{160\pi}{10^2} \approx 5\,\frac{m}{s^2}$

c) notatki od Ruth $(90°)$

Problem 3.18. A point mass moves in a circle of radius $R = 20$ cm with a constant linear acceleration $a_s = 5$ cm/s^2. After what time t from the start of the motion will the centripetal acceleration a_c be twice the value of the linear acceleration?



Data:

$R = 20cm$

$a_s = 5\,\frac{cm}{s^2}$

$$a_c \xrightarrow{t} 2a_s \qquad a_c = \frac{v^2}{r} \qquad v = a_s t$$

$$a_c = \frac{(a_s t)^2}{R}$$

$$2a_s = \frac{a_s^2 t^2}{R}$$

$$2\cancel{a_s} R = a_s^{\cancel{2}} t^2$$

$$\sqrt{\frac{2R}{a_s}} = t$$