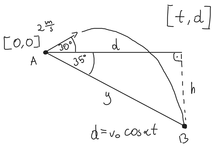
Problem 3.10. Standing on a slope inclined at an angle $\alpha = 35°$ to the horizontal, we throw a metal ball with an initial velocity $v_0 = 2$ m/s at an angle $\alpha = 30°$ towards the greatest descent of the slope. Where will the ball land (measured from the point of throw)? What will be the total time of its flight? Ignore air resistance.



$d = v_0 \cos\alpha\, t$

$v(t) = \left[v_0 \cos\alpha\, t \; ; \; v_0 \sin\alpha\, t - \frac{gt^2}{2} \right]$

$\tan(-35°) = \frac{h}{d} \Rightarrow -0{,}7 \approx \frac{h}{d} \Rightarrow$ $-0{,}7d = h$, $-h = 0{,}7d$

$-h = v_0 \sin\alpha\, t - \frac{gt^2}{2}$

$0{,}7d = v_0 \sin\alpha\, t - \frac{gt^2}{2}$

$0{,}7 v_0 \cos(30°) t = v_0 \sin(30°) t - \frac{gt^2}{2}$ huh?

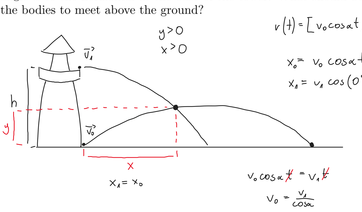
$0{,}7 \cdot 2 \cdot \frac{\sqrt{3}}{2} \cdot t = t - 5t^2$

$1{,}4 \cdot \frac{\sqrt{3}}{2} t - t + 5t^2 = 0$

$5t^2 - 0{,}21t = 0$ $\quad 5t = 0{,}21$

$t(5t - 0{,}21) = 0$ $\quad t = 0{,}042$

Problem 3.11. Two bodies are thrown simultaneously from 2 different points. One body is thrown with a velocity v_1 horizontally from a tower of height h, the other with a velocity v_0 at an angle α to the horizontal at the base of the tower. What should be the value of v_0 and α for the bodies to meet above the ground?



$y > 0$
$x > 0$

$v(t) = \left[v_0 \cos\alpha\, t \; ; \; v_0 \sin\alpha\, t - \frac{gt^2}{2} \right]$

$x_0 = v_0 \cos\alpha\, t$

$x_1 = v_1 \cos(0°) t \Rightarrow x_1 = v_1 t$

$v_0 > 0$
$v_1 > 0$

$\alpha \in (0°, 90°)$

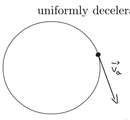
$x_1 = x_0$

$v_0 \cos\alpha\, \cancel{t} = v_1 \cancel{t}$

$v_0 = \frac{v_1}{\cos\alpha}$

$\cos\alpha \neq 0$
$\arccos(0) \neq \alpha$
$\alpha \neq 90°$

Problem 3.12. A flywheel initially making N revolutions per minute starts to slow down. Calculate the time after which the angular velocity of the wheel is halved, if the disc's motion is uniformly decelerated and the disc made x revolutions in this time.



$\omega(t) = \omega_0 - \epsilon t$

$\omega = \frac{2\pi}{T}$

$n(t) = \omega_0 t - \frac{\epsilon t^2}{2}$

$\omega_0 = \frac{N 2\pi}{60} \left[\frac{rad}{s} \right]$ $\quad t_0 = 0$

$\omega_1 = \frac{N\pi}{30}$ $\quad \omega_1 = \frac{\omega_0}{2}$; t_1

$N_{rpm} = N \frac{2\pi}{60} \frac{rad}{s}$

$n(t)$ revolutions made already

$\epsilon = \frac{\Delta\omega}{\Delta t} = \frac{\omega_0 - \frac{\omega_0}{2}}{t_1}$

$x = \omega_0 t_1 - \frac{\frac{\omega_0 - \frac{\omega_0}{2}}{\cancel{t_1}} t_1^{\cancel{2}}}{2}$

$x = \omega_0 t_1 - \frac{1}{2} t_1 \left(\omega_0 - \frac{\omega_0}{2} \right)$

$x = t_1 \left(\omega_0 - \frac{1}{2}\omega_0 - \frac{\omega_0}{4} \right)$

$\frac{x}{\frac{1}{4}\omega_0} = t_1$ $\quad \omega_0 \neq 0$

$\frac{4x}{\omega_0} = t_1$

Problem 3.13. Initially stationary, a disc of radius r is set into rotational motion. Find the angular velocity of the disc after time t_k, knowing that points on the rim of the disc covered a distance of $100\pi r$ during this time. Assume that the angular acceleration of the disc is constant.

$\dot{\omega}$ - const

$\omega_0 = 0$

$n(t) = \omega_0 t + \frac{\dot{\omega} t^2}{2}$

$\dot{\omega} = \frac{\Delta\omega}{\Delta t} = \frac{\frac{\Delta s}{r \Delta t}}{\Delta t} = \frac{\Delta s}{r(\Delta t)^2} = \frac{100\pi \cancel{r}}{\cancel{r} t_k^2}$

$n(t) = \frac{\dot{\omega} t^2}{2}$

$100\pi r = \frac{\frac{100\pi}{\cancel{r}} t_k^{\cancel{2}}}{2}$

$\cancel{100\pi r} = \cancel{50\pi}$

$r = \frac{1}{2}$

$\Delta\theta = \frac{\Delta s}{r}$

$\omega = \frac{\Delta\theta}{\Delta t}$

$\omega = \frac{\Delta s}{\frac{r}{\Delta t}}$

$\omega = \frac{\Delta s}{r \Delta t}$

$\omega = \frac{\Delta\theta}{\Delta t}$

$\dot{v} = r\omega$

$\dot{v} = \frac{1}{2}\omega$

$v^0 = \frac{1}{2} \cdot \frac{s}{\frac{1}{2} t_k} = \frac{s}{t_k}$

Problem 3.14. A point mass moves around a circle with a velocity $v = \beta t$, where $\beta = 0.50$ m/s^2. Find its acceleration and the angle it makes with the velocity vector after completing the first full rotation.

$v = \beta t$

$\beta = 0.5 \frac{m}{s^2}$

$V_d = \frac{v^2}{r}$

$V_d = \frac{(\beta t)^2}{r}$

$v = \beta t$

$v^2 = (\beta t)^2$

$V_d = \frac{\beta^2 t^2}{r}$

$V_d = \frac{\beta^{\cancel{2}} \cdot \frac{4\pi r}{\beta}}{\cancel{r}}$

$V_d = \frac{v^2}{r}$

$\boxed{2\pi v}$

$s(t) = \overset{0}{s_0} + \frac{\beta t^2}{2}$

$2\pi v = \frac{\beta t^2}{2} \quad | \cdot 2$

$4\pi v = \beta t^2$

$\frac{4\pi v}{\beta} = t^2$

$t = \sqrt{\frac{4\pi v}{\beta}}$

$V_d = 4\pi\beta$

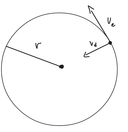
$a = \sqrt{\beta^2 + (4\pi\beta)^2}$

$a = \beta\sqrt{1 + (4\pi)^2}$

$\tan\alpha = 4\pi$

$\alpha \approx 1{,}5$ rad

Problem 3.15. Calculate the linear velocity of a point in the Earth's rotational motion (in a system tied to the centre of the Earth): (A) at the equator, (B) at a latitude of 48°24′. The radius of the Earth is 6378 km.



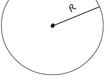
$r = 6378$ km $\quad t = 24$ h

$v = \frac{s}{t}$

$v = \frac{2\pi r}{24} = \frac{\pi r}{12} = \frac{6378 \cdot \pi}{12} \approx 1669{,}8$

$\cos(48°24') \cdot 6378 \cdot 2\pi = 26\,606$ km

$v = \frac{26606}{24} = 1108{,}58$

Problem 3.16. A particle covered half a circle of radius $R = 160$ m in a time $t = 10.0$ s with a constant value of tangential acceleration. Calculate over the time interval from 0 to t: (a) the magnitude of the average velocity vector; (b) the magnitude of the average acceleration vector; (c) the angle between these vectors.

$a_t = \dot{\omega} r$