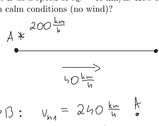
Problem 3.1. A helicopter travelling at a speed of $v_h = 200$ km/h goes from point A to point B and back in a time of $t = 1$ h 50 min. During the flight, there is a wind blowing from point A to point B at a speed of $v_w = 40$ km/h. How long will the helicopter take to complete the journey in calm conditions (no wind)?



A->B: $v_{n1} = 240 \frac{km}{h}$

B->A: $v_{n2} = 160 \frac{km}{h}$

$t_1 = \frac{s_1}{240}$

$t_2 = \frac{s_1}{160}$

A->B->A

t: 1h 50min

$\frac{s_1}{240} + \frac{s_1}{160} = 1\frac{50}{60}$

$\frac{160 s_1 + 240 s_1}{38400} = \frac{110}{60}$

$\frac{2s}{384} = \frac{11}{6}$

$12s = 4224$

$s = 352$ km

$V = \frac{S}{t}$

$s = vt$

$t = \frac{s}{v}$

$t_1 + t_2 = 1h\ 50min$

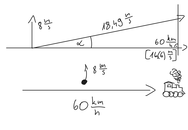
$s = 2s_1$

$t = \frac{352}{200} = 1,76h$

$105,6$ min

1h 45,6 min

Problem 3.2. A ball rolls inside a train car, perpendicular to the direction of the train's movement, at a speed of $v_1 = 8$ m/s relative to the train, which moves at a speed of $v_2 = 60$ km/h. What is the speed of the ball relative to the Earth in magnitude and direction?



relative to the earth:

$v_{rel} = \sqrt{8^2 + [16,6]^2} = 18,49 \frac{m}{s}$

$\cos\alpha = \frac{16,6}{18,49} = 0,9$

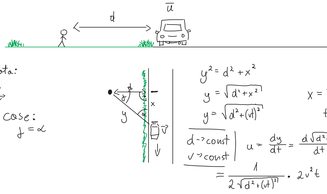
$\arccos(0,9) = \alpha$

$\alpha = 25,84°$

$\tan\alpha = \frac{8}{16,6} = 0,48$

$\arctan(0,48) = \alpha \approx 25,63°$

Problem 3.3. A tourist is at a certain distance from a road along which a car is travelling at speed v. Calculate the rate u at which the distance between the car and the tourist changes at the moment when the line connecting the car and the tourist forms an angle α with the direction of the road.



Data:
d
v
Case: $\alpha = \alpha$

$y^2 = d^2 + x^2$

$y = \sqrt{d^2 + x^2}$

$y = \sqrt{d^2 + (vt)^2}$

$x = vt$

$t_0 = 0$

lim $t \to 0$

d -> const
v -> const

$u = \frac{dy}{dt} = \frac{d\sqrt{d^2 + (vt)^2}}{dt} =$

$= \frac{1}{2\sqrt{d^2 + (vt)^2}} \cdot 2v^2 t = \frac{2v^2 t}{2\sqrt{d^2 + v^2 t^2}}$

Problem 3.4. A body, falling freely, has a speed of $v_A = 29.43$ m/s at point A and a speed of $v_B = 49.05$ m/s at point B. How long does the body take to travel this path and what is the length of segment AB?

$g = 9,81$

free fall

$\vec{v}(t) = [v_0 \cos\alpha,\ v_0 \sin\alpha - gt]$

$\vec{r}(t) = [v_0 \cos\alpha\, t,\ v_0 \sin\alpha\, t - \frac{gt^2}{2}]$

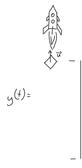
$49,05 = 29,43 + -9,81t$

$19,62 = 9,81t$

$2s = t$

$d = 78,48m$

Problem 3.5. A rocket, ascending vertically upwards, loses an unnecessary fuel tank at a height of h when it has a velocity u. Find the time t, after which the tank will fall to the ground. Ignore air resistance.



$v(t) = [v_0 \cos\alpha,\ v_0 \sin\alpha - gt]$

$r(t) = [v_0 \cos\alpha\, t,\ v_0 \sin\alpha\, t - \frac{gt^2}{2}]$

$-h = u \cdot \sin(-90°) - \frac{g \cdot t^2}{2}$

$t^2 = \frac{2(h-u)}{g}$

$-h = -u - \frac{1}{2} g t^2$

$t = \sqrt{\frac{2(h-u)}{g}}$???

$h = u + \frac{1}{2} g t^2$

$h - u = \frac{1}{2} g t^2$

Problem 3.6. Body A is thrown vertically upwards with a velocity $v_0 = 50$ m/s. After a time $t = 1$ s, body B is thrown with the same velocity in the same direction. When, where, and at what velocity will these bodies meet?

$r(t) = [v_0 \cos\alpha\, t;\ v_0 \sin\alpha\, t - \frac{gt^2}{2}]$ $\sin\alpha = 1$

$50t - \frac{gt^2}{2} = 50(t+1) - \frac{g(t+1)^2}{2}$ $\mid 2$

$100t - gt^2 = 100(t+1) - g(t+1)^2$

$100t - 100t^2 = 100(t+1) - 10(t+1)^2$ $\mid :10$

$10t - t^2 = 10(t+1) - (t+1)^2$

$10t - t^2 = 10t + 10 - t^2 - 2t - 1$

$2t = 9$

$t = 4,5$ s

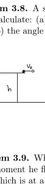
$v_A = 50 - 10 \cdot 4,5 = 5 \frac{m}{s}$

$v_B = 50 - 10 \cdot 3,5 = 15 \frac{m}{s}$

$h = 50 \cdot 4,5 - \frac{5 \cdot 4,5^2}{2}$

$h = 123,75$ m

Problem 3.7. An object falls from a height H with an initial velocity v_0 directed downwards. After n seconds, the object is at a height h above the Earth. What distance will this body cover in the next, $(n+1)$th second?



$h(t) = v_0 t - \frac{gt^2}{2}$

$h(t+1) = v_0(t+1) - \frac{g(t+1)^2}{2}$

$v = \frac{s}{t}$

$vt = s$

???

$v_0 n - \frac{gn^2}{2} = v_0(n+1) - \frac{g(n+1)^2}{2}$ $\mid 2$

$2v_0 n - gn^2 = 2v_0(n+1) - g(n+1)^2$

$2v_0 n - gn^2 = 2v_0(n+1) - g(n+1)^2$

$2v_0 n - gn^2 = 2v_0(n+1) - g(n+1)^2$

$2v_0 n - gn^2 = 2v_0 n + 2v_0 - gn^2 - 2gn - g$

$0 = 2v_0 - 2gn - g$

$2gn = 2v_0 - g$ $\mid : 2g$

$n = \frac{2v_0 - g}{2g}$

$[H-h](t) = h_{t(t+1)}$

$v(t) = [v_0 \cos\alpha;\ v_0 \sin\alpha - \frac{gt^2}{2}]$

$h_t = v_0 \cdot \sin(-30°) \cdot \left(\frac{2v_0 - g}{2g} + 1\right) - \frac{gt^2}{2}$

Problem 3.8. A stone is thrown from a table of height h horizontally with an initial velocity v_0. Calculate: (a) the components of the velocity vector at the moment the stone hits the ground; (b) the angle between the velocity vector and the vertical at the moment of impact.

$v_t = [v_0;\ -gt]$

$\tan\alpha = \frac{-gt}{v_0}$

$\cot\alpha\left(\frac{-gt}{v_0}\right) = \alpha$

Problem 3.9. What horizontal velocity v_1 must a pilot have at a height h above the tracks, at the moment he flies over point A, so that a dropped bomb hits a train escaping at a velocity v_2, which is at a distance d from A (the plane and the train move in the same direction).

$s = vt$

$x = v_2 \sqrt{\frac{2h}{g}}$

$v(t) = [v_0 \cos\alpha\, t,\ v_0 \sin\alpha\, t - \frac{gt^2}{2}]$

$-h = v_0 t \sin\alpha - \frac{gt^2}{2}$

$v = \frac{s}{t}$

$-h = -\frac{gt^2}{2}$

$h = \frac{gt^2}{2}$ $\mid 2$

$2h = gt^2$

$\sqrt{\frac{2h}{g}} = t$

$v_2 = \frac{d+x}{t}$

$\frac{d + \sqrt{\frac{2h}{g}}}{\sqrt{\frac{2h}{g}}} \cdot \frac{\sqrt{\frac{2}{g}}}{\sqrt{\frac{2}{g}}} = \frac{d\sqrt{\frac{2h}{g}} + v_2\left(\frac{2h}{g}\right)}{\frac{2h}{g}} =$

$= \frac{g}{2h}\left(d\sqrt{\frac{2h}{g}} + v_2 \frac{2h}{g}\right) =$

$= \frac{dg}{2h}\sqrt{\frac{2h}{g}} + v_2 = d\sqrt{\frac{g^2}{(2h)^2}\frac{2h}{g}} + v_2 =$

$d\sqrt{\frac{g}{2h}} + v_2$