Problem 4.1. A body of mass $m = 100$ g was thrown vertically upwards from a height $h = 2$ m with an initial velocity $v_0 = 2$ m/s. Calculate the kinetic energy of this body at a height of 1 m.

$$\frac{mv_0^2}{2} + mgh = E_k + mgh'$$

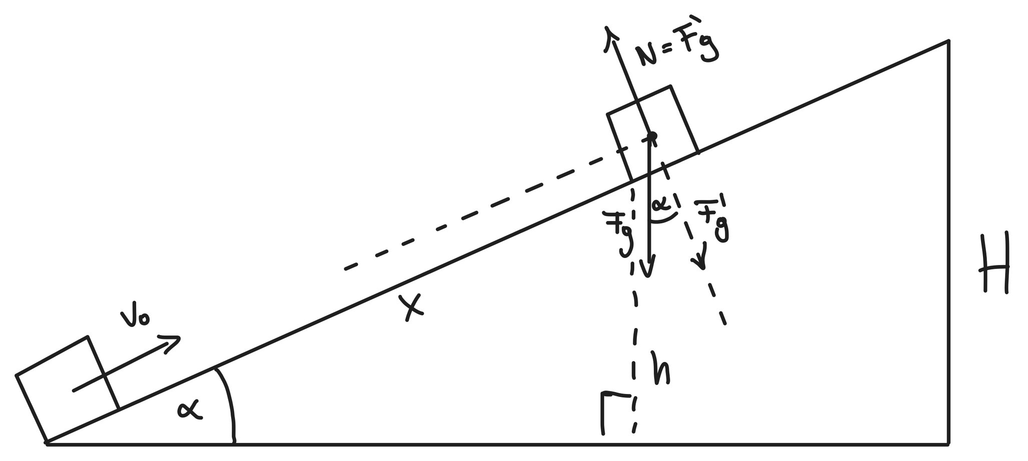
$$\frac{0{,}1 \cdot 2^2}{2} + 0{,}1 \cdot 10 \cdot 2 = E_k + 0{,}1 \cdot 10 \cdot 1$$

$$0{,}2 + 2 = E_k + 1$$

$$1{,}2 = E_k$$

Problem 4.2. Calculate at what height h will a block stop, pushed upwards on a slope with an angle of inclination $\alpha = 30°$ if the coefficient of friction of the block on the slope is $f = 0.24$. Assume that the block has small dimensions compared to the slope.

$v = 5\frac{m}{s}$



$x = ???$

$$\sin\alpha = \frac{h}{x}$$

$$x\sin\alpha = h$$

$$x = \frac{h}{\sin\alpha}$$

$F_g = mg \quad | \quad F_g' = mg\cos\alpha$

$E_{tt} = fNx$

$$E_k + E_p = E_k' + E_p' + E_{tt}$$

$$\frac{mv^2}{2} = mgh + fmg\cos\alpha\, x$$

$$mv^2 = 2mgh + 2fmg\cos\alpha\, x$$

$$mv^2 = 2mgh + 2fmg\cot\alpha\, h$$

$$v^2 = 2gh + 2fgh\cot\alpha$$

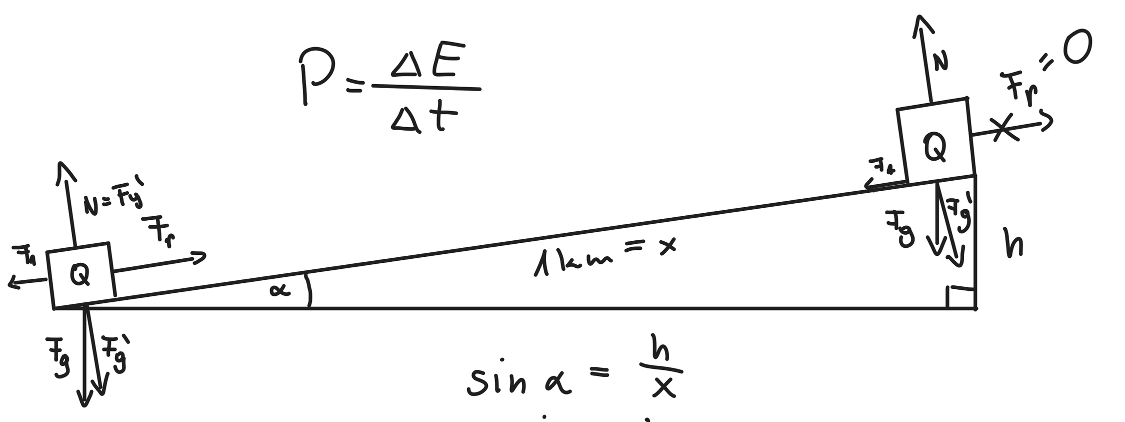
$$v^2 = h(2g + 2fg\cot\alpha)$$

$$h = \frac{v^2}{2g(1 + f\cot\alpha)}$$

Problem 4.3. A horse pulls a sled up a hill which is inclined at an angle $\alpha = 10°$ to the horizontal. The sled, weighing $Q = 6200$ N, moves at a constant speed, covering 1 km in 9 minutes. With what power does the horse work pulling the sled? The coefficient of friction is $f = 0.05$. How will the required power from the horse change if the coefficient of friction doubles?

ma

$$P = \frac{\Delta E}{\Delta t}$$



$t = 9\text{ min} = 540\text{s}$

$Q = 6200\text{N}$

$f = 0.05$

620 kg

$$\sin\alpha = \frac{h}{x}$$

$$x\sin\alpha = h$$

$$1000 \cdot \sin 10° = h$$

$$h \approx 0.173648 \cdot 1000$$

$$E_{k0} = \frac{mv^2}{2}$$

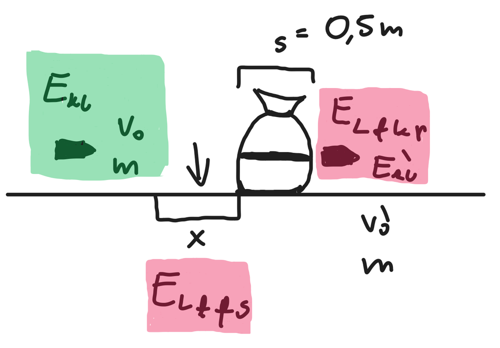
$$E_{kk} = \frac{mv^2}{2} + mgh + mg\cos\alpha f$$

$$\Delta E = mgh + mg\cos\alpha f$$

$$6200 \cdot 0.173648 + 6200 \cdot 0.98481 \cdot 0.05$$

$P = 1994.3\text{W}$

Problem 4.4. A bullet of mass $m = 20$ g, moving at a speed $v = 600$ m/s, passes through a sandbag of mass $M = 150$ kg, causing the bag to move by $x = 20$ cm. The bullet's speed decreases by 5% after exiting the bag. Calculate the coefficient of friction of the bag on the ground, knowing that the friction of the bullet through sand is $T_{kp} = 300$ N (in this case, the force of resistance is proportional to speed), and the bag has a transverse size $s = 0.5$ m.



$x = 0{,}2\text{m}$

$v_0 - 5\% v_0 = v_0'$

$v_0' = 570\frac{m}{s}$

???

$$E_{k0} = E_{k0}' + E_{t1} + E_{t2}$$

$$\frac{mv_0^2}{2} = \frac{m(v_0 - 5\% v_0)^2}{2} + (m+M)gfx + T_{kp}s$$

$$mv_0^2 = m(v_0 - 5\% v_0)^2 + 2(m+M)gfx + 2T_{kp}s$$

$$mv_0^2 - mv_0'^2 - 2T_{kp}s = 2(m+M)gfx$$

$$f = \frac{mv_0^2 - mv_0'^2 - 2T_{kp}s}{2(m+M)gx}$$

$$f = \frac{0.02 \cdot 600^2 - 0.02 \cdot 570^2 - 2 \cdot 300 \cdot 0{,}5}{2(0.02 + 150) \cdot 10 \cdot 0{,}2}$$

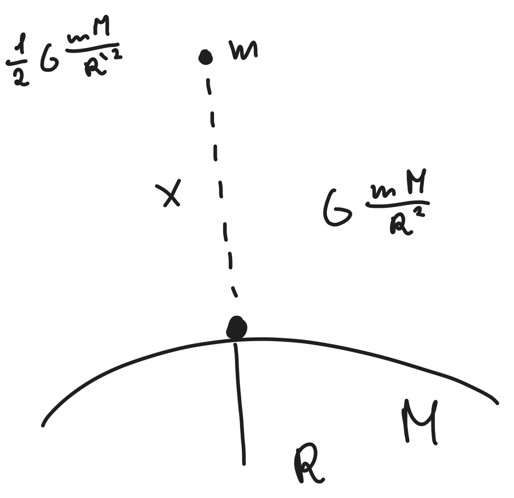
$$f = \frac{702 - 300}{600.08} = \frac{402}{600.08} \approx 0.6699$$

$E_{t1} = T_1 s$

$E_{t2} = Nfs$

$N = mg$

Problem 4.5. Calculate at what height above the Earth's surface the weight of a body is twice less than its weight on the surface of the Earth.



$$\frac{1}{2}G\frac{mM}{R^2} \qquad G\frac{mM}{R^2}$$

$$F = G\frac{mM}{R^2}$$

$$\frac{1}{2}G\frac{mM}{R^2} = G\frac{mM}{(R+x)^2}$$

$$\frac{1}{2R^2} = \frac{1}{(R+x)^2}$$

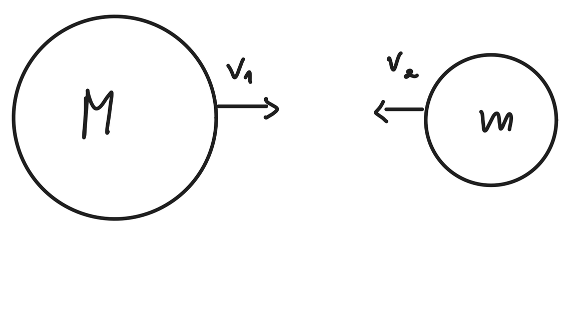
$$2R^2 = (R+x)^2$$

$$\sqrt{2}R = R + x$$

$$R(\sqrt{2} - 1) = x$$

Problem 4.6. At what distance R_S from the center of the Earth should a satellite orbit to remain constantly above the same point on the Earth's surface? Express the formula for the satellite's orbit radius in terms of the Earth's radius R_Z, the gravitational acceleration g, and the Earth's rotation period T_Z.

Problem 4.8. Two balls of masses $M = 5$ kg and $m = 3$ kg, moving at speeds $v_1 = 12$ cm/s and $v_2 = 4$ cm/s, collide head-on. Calculate the speeds of the balls after the collision: (a) in the case of inelastic balls, (b) in the case of perfectly elastic balls.



$$\frac{Mv_1^2}{2} + \frac{mv_2^2}{2} = \frac{(m+M)v^2}{2}$$

$$Mv_1 - mv_2 = (m+M)v$$

$$5 \cdot 12 - 3 \cdot 4 = 8v$$

$$\frac{60 - 12}{8} = v$$

$$6 = v$$