Fizyka

Set 1.

11) $\vec{a}=[4,3,-2]$ $\vec{b}=[4,-5,9]$

$\vec{a}\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|\cdot\cos\alpha$

$\cos\alpha=\frac{\vec{a}\vec{b}}{|\vec{a}||\vec{b}|} \Rightarrow \cos\alpha=\frac{16-15-18}{\sqrt{3538}}=\frac{-17}{59.5}$

$|\vec{a}|=\sqrt{16+9+4}=\sqrt{29}$

$|\vec{b}|=\sqrt{16+25+81}=\sqrt{122}$

$\alpha\approx 106.6$

13) $\vec{c}=[1,2,3], \vec{d}=[0,2,1]$

$\vec{c}\times\vec{d}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&2&3\\0&2&1\end{vmatrix}=\begin{matrix}\hat{i}(2-6)\\\hat{j}(0-1)\\\hat{k}(2-0)\end{matrix}=[-4,-1,2]$

$\vec{g}=[0,2,4], \vec{h}=[4,0,2]$

$\vec{g}\times\vec{h}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\0&2&4\\4&0&2\end{vmatrix}=\begin{matrix}\hat{i}(4-0)\\\hat{j}(16-0)\\\hat{k}(0-8)\end{matrix}=[4,16,-8]$

Set 3

$\tan\alpha=\frac{v_1}{v_2}$

Problem 3.5. A rocket, ascending vertically upwards, loses an unnecessary fuel tank at a height of h when it has a velocity u. Find the time t, after which the tank will fall to the ground. Ignore air resistance.

$t=???$

$r(t)=\left[v_0\cos\alpha t;\ v_0\sin\alpha t-\frac{gt^2}{2}\right]$

$-h=ut-\frac{10t^2}{2}$

$-h=ut-5t^2$

$0=-5t^2+ut+h$

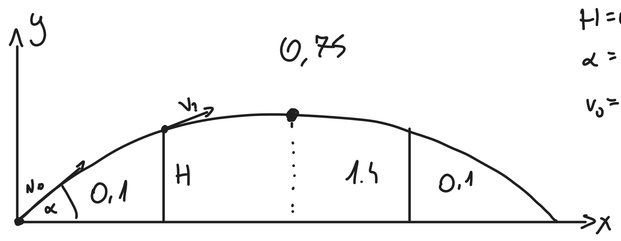
$\Delta=u^2+20h$

$\sqrt{\Delta}=\sqrt{u^2+20h}$

$t=\frac{-u\pm\sqrt{u^2+20h}}{-20}$

Set 2.

A jet of water leaves a water canon at an angle α=30° to the horizontal with a speed of $15\frac{m}{s}$. After what time will it reach the height H=0.7 m from the level it leaves the canon? What velocity does it have at this point? Which maximum height will it reach and after what time? How far from the canon does the water hit the ground and after what time?



$H=0,7$

$\alpha=30°$

$v_0=15\frac{m}{s}$

$v(t)=[v_0\cos\alpha,\ v_0\sin\alpha-gt]$

$r(t)=\left[v_0\cos\alpha t,\ v_0\sin\alpha t-\frac{gt^2}{2}\right]$

x y

$H=v_0\sin\alpha t-\frac{gt^2}{2}$

$0,7=15\sin(30°)t-\frac{gt^2}{2}$

$1,4=15t-10t^2$

$0=-10t^2+15t-1,4$

$\Delta=225-56=169$

$\sqrt{\Delta}=13$

$t=\frac{-15\pm13}{-20}$

$x_1=\frac{-2}{-20}=\frac{1}{10}=0.1$

$x_2=\frac{-28}{-20}=1,4$

$\vec{v}(t)=\left[\frac{\sqrt{3}}{2}\cdot15,\ 15\cdot\frac{1}{2}-10\cdot0.1\right]=\left[7,5\sqrt{3},\ 6,5\right]$

$|\vec{v}(t)|=\sqrt{(7,5\sqrt{3})^2+(6,5)^2}=14,526$

x_{max}

$x_{max}=15\cdot\frac{\sqrt{3}}{2}\cdot1,5$

$x_{max}=11,25\sqrt{3}$

y_{max}

$0=15\cdot\frac{1}{2}-10\cdot t$

$0=7,5-10t$

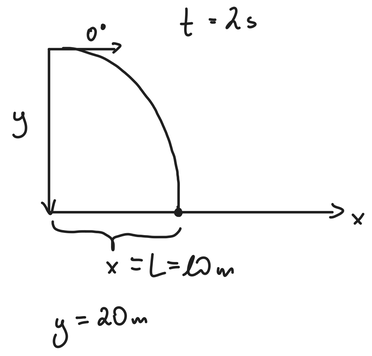
$7,5=10t$

$0.75=t$

$15\cdot\frac{1}{2}\cdot0,75-\frac{10\cdot(0.75)^2}{2}$

$y_{max}=5.625-2.8125=2.8125$

A body was launched horizontally from the top of a tower and it reached the bottom after time t = 2s at a distance of L = 10m from the base of the tower. Calculate the initial speed of the body, and speed with which is hit the ground. What was the height of the tower?



$t=2s$

$y=20m$

$r(t)=\left[v_0\cos\alpha t;\ v_0\sin\alpha-\frac{gt^2}{2}\right]$

$10=v_0\cos(0°)\cdot2$

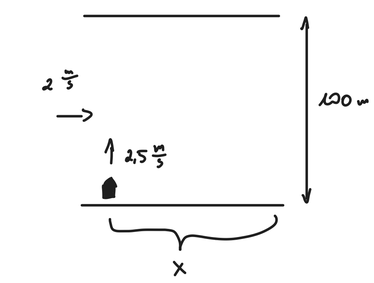
$5=v_0\ [\frac{m}{s}]$

$y=5\sin(0°)-\frac{10\cdot2^2}{2}$

0

$y=20$

A boat crosses a river that is 100 m wide with a speed of $v_1 = 2.5$ m/s in a direction perpendicular to the riverbank, which flows with a speed of $v_2 = 2$ m/s. By how many meters will the boat be carried downstream upon landing?



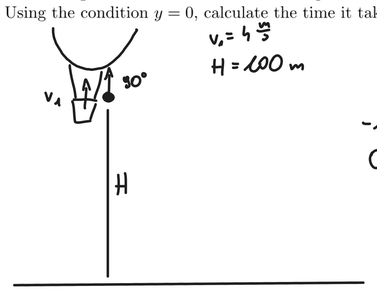
$v=\frac{s}{t}$ -> $vt=s$ -> $\frac{s}{v}=t$

$\frac{100}{2.5}=40s$

$x=2\frac{m}{s}\cdot40s$

$x=80m$

From a balloon ascending upwards at a speed of $v_1 = 4$ m/s, a sack of sand is dropped when the balloon is at a height of $H = 100$ m. Write the equation for the vertical and horizontal coordinates of the sack. Using the condition $y = 0$, calculate the time it takes for the sack to fall.



$v_0=4\frac{m}{s}$

$H=100$ m

$r(t)=\left[v_0\cos\alpha t;\ v_0\sin\alpha t-\frac{gt^2}{2}\right]$

$-100=v_0t-\frac{gt^2}{2}$

$0=-5t^2+4t+100$

$t=4.89$